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TWO UNITS IN NINTH YEAR MATHEMATICS

Submitted by

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(A.B. Florida, 1938)

In Partial Fulfillment of Requirements for

the Degree of Master of Education

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CHAPTER I

INTRODUCTION

A. Statement of the Problem.--The course in elementary algebra, as outlined in Booker Washington High School in Miami, Florida, is designed primarily to meet the needs of pupils in the college preparatory curriculum. The present study attempts to enrich such a course by the introduction of subject matter which will appeal to the slow learner, as well as those not interested in the academic curriculum. This study is a plan to devise a course in ninth year that will meet both the needs of pupils in the general and in the college preparatory curricula; to set up a program that will arouse the brilliant pupils and yet not be too difficult for the average pupil to comprehend.

The problem is to construct two units in detail, including more practical social application than is offered in the average text book in elementary algebra. As soon as these units have been properly administered others will be completed in detail for the course.

Subject matter revised.-- There is to be a complete revision of subject matter. Only topics having a definite meaning for the pupils will be used, these will be centered around socio-economic topics rather than algebraic ones. These topics are chosen because they are items with which the pupils in all walks of life must come in contact with. The principles of algebra will be woven in as incidentals to his successful participa-

tions in everyday living. He is already aware of the part that insurance plays in our lives today. He does not doubt the need of a proper background to compute personal accounts. He has already started saving accounts--so is vitally interested in good investments and how they accumulate. Likewise, he can be led into discovering for himself the need of certain algebraic principles or laws that he has been utilizing all along.

Optional related material.-- The brighter and more interested pupils will cover this supplementary (marked with *) material as extra work. The maximum course of study will never be as definite as the minimum. The pupils will be told of the two levels of work. The work not marked * will be the D-C level. These must all be tried by everyone. The A-B level will include the lower level plus the supplementary marked exercises. In the lower level the grade will be determined by the quality of work done, but in the upper level the grade will be determined by not only the quality of work done, but in the upper level the grade will be determined by not only the quality but also the quantity.

B. General Trend Toward Revision of Mathematics for Ninth Year.-- Over a period of years the revision of mathematics for ninth year has been great. The algebra of our first high school was for the Latin Grammar School, thus it had for its sole purpose the preparation of boys for entrance to college. The transition from one influence of one educational philosophy to that of another has been characterized by a somewhat deliberate effort at reform under the persistent demands of social changes.

As a result of the movement toward general mathematics in Junior High Schools, we find that the general mathematics courses have been

changed from the formal compartmental arrangement to studies with concrete beginnings, less scientific rigor, more developmental and explanatory material, more practical exercises, better psychological development, more provision for individual difference and better fusion of related topics.

In a curriculum pointed toward the goals of genuine social and individual competence, mathematics will play an important part. In the ninth year, in particular, revision of mathematical instructions has been very pronounced. There has been an injection of new significant materials, a deletion of many traditional practices and units of subject matter, a reallocation of concepts and skills, a new emphasis on the abstraction of mathematical processes, and a vitalization of instructional techniques.

The changing character of the secondary-school population has brought into existence in the last few years one type of course for ninth year mathematics which is radically different from the traditional courses in algebra, both in aim and content. There are certain students in the secondary schools who have neither the capacity nor the inclination to pursue the regular sequence of mathematical courses in the high school and who do not expect to go to college or follow work for which this course would be a prerequisite. It is for such students that these revisions have been designed. They consist mainly of useful applications and extensions of the work of the earlier grades rather than the development of a new mathematical theory. Principles of logical organization or subject matter are largely subordinated to considerations of interest and social and economic usefulness.

In a study made of trends in the reorganization of Junior High School Mathematics, Karp[1] reveals the necessity for the reorganization of the traditional mathematics courses in the entire Junior High School with special reference to the ninth year. It is also revealed here by statistics that the pupils have encountered more difficulties in the study of mathematics than with any other subject. This apparent failure is due either to a faulty organization and improper choice of the subject matter which is to be presented or to faulty methods of teaching.

Even though larger high schools are using the plan of the multiple curricula, there is still the temptation to place the boy or girl in the one that is most convenient for the administration. It is such pupils as these that are misfits in courses. If a course is reorganized, or organized, so that the important facts are taught and are in such a way that no pupil will suffer a result of being thrown in.

The fact that it is the job of the teacher to use foresight in revision is best described in the following editorial by W. D. Reeve, "A new day in mathematics ought to bring forth a generation of teachers who can set up their own objectives, select the content material best fitted to their realization, and with proper methods of teaching organize an evaluation program that will enable teachers to find out whether their aims can be realized."[2]

1/ A. L. Karp, "Trends in the Reorganization of Junior High School Mathematics", Unpublished Masters Thesis. B. U. School of Education 1941.

2/ W. D. Reeve, "Modern Curriculum Problems in the Teaching of Mathematics in the Secondary Schools", The Mathematics Teacher (32:118)

The attitude before this way resembled the attitude previous to World War I toward mathematics. The trend was to discourage pupils from taking algebra and higher mathematics. To this end, many states, schools and school systems reduced or eliminated mathematical requirements for graduation.

Teachers of mathematics protest.-- Teachers of mathematics were acutely aware of this trend and were loud in their protest. Efforts were made to enrich the mathematics curriculum by correlating it with various subjects, especially sciences. In all, the subject was presented in a more concrete fashion.

Among those raising their voices in demand for improvement were such educators as Myers, Breslich, Reeve, Schorling, Clarke and many others. Ninth year mathematics then went through a name-calling period. The concensus of opinion, however, was that there should be a closer correlation of algebra with other subjects in the mathematics curriculum. These subjects included: arithmetic, informal geometry and numerical trigonometry.

Revision needed.-- Revision of such a course should help in more ways than one. The pupil very often comes into the course with the preconceived notion that the subject is to be difficult and regardless of the efforts of the teacher to give proper introduction, and to improve the methods of instruction this idea oftimes persists.

These pupils must be led toward discovering that the fundamental processes and symbols are the same; that algebra is merely a new and different approach to the study of quantative relationship; that although proper study begins in the ninth grade he has been exposed to many of the concepts in the seventh and eighth grades.

The chances for possible and desirable applications of mathematical principles are growing from year to year. Long ago the tendency prevailed to perform algebraic process mechanically.

In relation to these rules and processes in algebra, Breslich is of the opinion that, "they do not seem to be learned but only memorized thus are not remembered. In years before each war, high school mathematics received much adverse criticisms from educators everywhere."[1/]

If we believe that algebraic processes are mostly rules developed for automatic response by drill then "any one can teach mathematics." If not, there is need for the teacher to have a sufficient background in mathematics and its application to the pupil and his everyday needs. She will need skills in the presentation of the subject matter as well as in the subject matter itself.

1/ E. R. Breslich, "The Influence of the War on the Teaching of Mathematics", ibid, (38:291)

C. Criteria and the Selection of Topics.-- The following criteria were used in the selection of subject matter:

I. Some of the materials in mathematics should be selected because they have social values for pupils.

II. Some of the materials for this course in mathematics should be selected because they are essential to the child's understanding of the fundamental operation of mathematics.

III. Some of the materials for this course in mathematics should be selected because they are vital and timely, therefore, are pertinent to the age.

IV. Some of the materials for this course in mathematics should be selected because they have immediate rather than deferred values.

V. Some of the materials for this course in mathematics should be selected because they insure mathematical competency for the ordinary affairs of life.

These five criteria were taken from the following sources:

Number I Fundamentals of Secondary School Teaching by Roy O. Billett

II Ninth-year Mathematics by Ernest R. Breslich

III Teaching of Secondary Mathematics by C.H. Butler & F.L. Wren

IV Teaching of Secondary Mathematics by C.H. Butler & F.L. Wren

V Post War Commission on the Reorganization of Mathematics in the Secondary Schools

There were four stages in the selection of the topics:1/

The first stage was the establishment of criteria for the selection of the material.

1/ R.O. Billett Fundamentals of Secondary School Teaching, Houghton Mifflin Company, Boston, Mass. P. 171

The second stage was a discovery through analysis of the subject matter available.

In the third stage the topics which had been discovered through analysis were checked against the five criteria.

In the fourth stage the topics which measured up to one or more of the five criteria were subjected to further selection by checking recent textbooks, periodicals, and research articles pertaining to the topics in question.

Topics previously covered in course.-- The following topics that were previously taught in this course were subjected to the criteria as shown in Table I.

a. Formula and graphical representation
b. Literal numbers
c. Positive and negative numbers
d. Simple equations
e. Factoring
f. Numerical trigonometry
g. Ratio, proportion, and variation
h. Quadratic equation
i. Complex fractions
j. Fractional equations
k. Simultaneous equations
l. Simple equations with two unknowns
m. Radicals
n. Equations involving radicals
o. Trigonometric ratio

Through the reading of periodicals and other current literature, the following were checked most favorably with the criteria:

a. Formulas and graphical representation
b. Literal numbers
c. Positive and negative numbers
d. Simple equations
e. Factoring

Supplementary material.-- The following topics will be taught only as supplementary:

a. Numerical Trigonometry
b. Ratio, proportion, and variation
c. Quadratic equations
d. Complex fractions
e. Fractional equations
f. Simultaneous equations
g. Simple equation with two unknowns

Elimination of unnecessary topics.-- There will be an elimination of unnecessary topics previously taught in this course:

a. Radicals
b. Equations involving radicals
c. Trigonometric ratios
d. Solving equations with two or more unknowns
e. Time, rate, distance and mixture problems

Addition of new material.-- The following will be worked in the new course of study at the end of the other topics:

a. Investments
b. Use of slide rule
c. Elementary statistics

TABLE I

HOW TOPICS WERE SELECTED

Criteria

I. Social Values

II. Fundamental Processes

III. Vital and Timely

IV. Immediate Values

V. Competency for Ordinary Affairs of Life

Topics	I	II	III	IV	V	Status
a. Formulas and Graphs	+	+	+	+	+	Included
b. Literal Numbers	+	+	+	+	+	Included
c. Positive and Negative Numbers	+	+	+	+	+	Included
d. Simple Equations	+	+	+	+	+	Included
e. Factoring	-	+	+	-	+	Included
f. Num. Trig.	+	-	+	+	+	Supplementary
g. Quadratics	-	+	-	+	+	Supplementary
i. Complex Fractions	-	-	-	-	+	Supplementary
j. Fractional Equations	-	-	-	-	+	Supplementary
k. Simultaneous Equations	-	-	-	+	+	Supplementary
l. Simple Equations - Two Unknowns	-	-	-	-	-	Eliminated
m. Radicals	-	-	-	-	-	Eliminated
n. Equations with Radicals	-	-	-	-	-	Eliminated
o. Trignometric ratios	-	-	-	-	-	Eliminated

Note: The Topic checked (+) plus measures favorable with the criterion; the Topic checked (-) minus measures unfavorably with the criterion.

CHAPTER II

METHOD OF PROCEDURE AND SOURCES OF DATA

The problem here is to construct these units so that they will be broad and elastic enough to provide for the individual differences as they may appear in this group of slow learners.

The principles followed in organizing these units are those advocated by Professor Roy O. Billett of Boston University[1/], and the organization of the study is based on Research methods as outlined by Waples and Tyler[2/].

It is not intended here that the text book will be abolished. When introducing a topic it is preferred to follow the textbook as close as is possible for many of the developments. Many times textbooks used are poor and out-of-date, but if it is used the pupil will not get the idea that there is disagreement between teacher and author, and aside from that he will have something to study when not under the direct supervision of the teacher. The work contained in the study guide is to be completed in the classroom but that is not to make the pupil feel that his text is not to be used nor relied upon. He is to understand that supplementary texts are to be used also. By so doing it is hoped to awaken in him the respect for mathematical laws and principles and to watch for a certain uniformity even in different interpretations. In the selection of these exercises the following sources were used:

1/ Roy O. Billett, Fundamentals of Secondary School Teaching, Houghton Mifflin Company, Boston, 1940.

2/ Douglass Waples and R. W. Tyler, Research Methods and Teachers Problems, The MacMillan Company, New York, 1930.

Books

E. R. Breslich, Ninth Year Mathematics

A. L. Cowan, Consumer Mathematics

E. L. Carpenter, E. L. Edgerton, and P. A. Carpenter, General Mathematics III

Nelson, Jacobs and Burrough, Everyday Problems in Mathematics

Sutton and Lennes, Economic Mathematics

J. A. Swenson, Integrated Mathematics

Theissen and McCoy, First Course in Algebra

Periodicals

The Mathematics Teacher

School Science and Mathematics

The Yearbooks for the National Council of Teachers of Mathematics

Suggested Courses of Study

State of Vermont

State of Kansas

State of New Hampshire

Unpublished and Published Theses

"Trends in the Reorganization of Junior High School Mathematics", A. L. Karp, Boston University School of Education

"Four Units in Solid Geometry", Margaret Bailey, Boston University School of Education

"Two Units in General Mathematics", Alexander Carter Boston University School of Education

"Insurance, An Informational Unit for Junior High School Mathematics", Gertrude B. Gallagher, Boston University School of Education

"Changes in Content of Elementary Algebra", Amy O. Chateuneuf

"Objectives in Ninth Grade Mathematics", W. M. Willette

Professional Books

C. H. Butler and F. L. Wren, The Teaching of Secondary Mathematics

E. R. Breslich, Problems in Teaching Secondary School Mathematics

J. W. A. Young, The Teaching of Mathematics Monograph

E. S. Lide, Instructions in Mathematics

CHAPTER III

UNIT ORGANIZATION OF THE TOPIC, ACCOUNTS

A. General Statement of the Unit.-- One should be able to understand simple statements of both commercial and social accounts. Man, today more than ever, needs the help of mathematics in his daily life in occupations or avocations. Accounts are a necessity in everyday living as he must make budgets, plan savings, pay bills and make investments and purchases.

Objectives: This unit on Accounts was built with the following objectives in sight:

1. To give the pupil a more accurate and ready working knowledge of the four fundamental operations.
2. To teach the value of the use of literal representation.
3. To teach the fundamental knowledge of the use of the equation, graph and formula, so as to give the student a working knowledge of these, so that he may be able to apply them to problems of everyday life.
4. To introduce algebraic technique in order to connect fundamental operations of algebra with arithmetical process.
5. To introduce the pupil to the use of positive and negative numbers in everyday life.

B. The Pupils.-- The following units and unit assignments were planned for 33 ninth grade pupils in a course in elementary algebra in a high school designed primarily for Negroes.

In chronological ages the pupils range from 13 years, 5 months to 17 years, 8 months.

In intelligence quotients the pupils range from 70 to 145.

In reading ability the pupils range from grade level 6.5 to 10.8.

Both parents of each of 13 pupils were born in Nassau, Bahamas. One parent of each of six other pupils were also born in the Bahamas. Both parents of each of eleven were born in the southern states. Others are from such islands as Cuba, Jamaica and other West Indian Islands. In most homes, poor English is spoken. Practically, all parents are unskilled laborers working in service as maids, janitors, gardeners, street cleaners or laundrymen. However, there are three children whose parents are professional workers. The schools and homes are located in the segregated slums of the city. Most of the parents of these pupils possess only an elementary education, if any at all.

The Time: Tentative time alloted for the completing of this unit is five weeks, five times a week.

C. Detailed Description of the Unit.-- This unit will be developed in the following manner:

There will be an introduction to the topic accounts involving first the process of buying and selling. As an outgrowth from the ideas of profit and loss will grow the idea of oppositeness. There follows then a detailed study of the fundamental operations in regard to directed numbers. The art of budgeting and income will then be discussed again mentioning such opposite quantities as assets and liabilities but stressing the different forms of graphical representation. The evaluative activities will be in the form of mastery tests.

D. Delimitation of the Unit.--

1. To understand principle of accounts one needs to understand fundamental operations.

2. The fundamental processes in all mathematics are the same.

3. To read certain parts of textbooks, magazines and newspapers intelligently one needs to understand opposite quantities.

4. To add a negative number is equivalent to subtracting the corresponding positive number.

5. In mathematics the plus and minus signs indicate opposite numbers.

6. One should be able to handle these numbers dealing with signs as well as numerical values only.

7. Subtracting a negative number is equivalent to adding the corresponding positive number.

8. One must understand the principles of buying and selling in order to be able to buy wisely.

9. Simple graphs commonly used: the bar graph; the circle graph; the curved-line graph; the broken-line graph; and the pictogram.

10. The circle graph is often used to picture the relation of the parts to the whole and to each other, for instance, the percentage of the total family income set aside for food, clothing, shelter, savings, insurance, recreation and amusement.

11. The bar graph is often used to picture a situation as it is at a given time, for instance, to compare the number of pupils belonging to the different clubs in the school in October of a certain year.

12. The broken-line graph is often used to picture a series of situations at success intervals of time, for instance, the number of members in a given club in each of five or ten success years.

13. The pictogram can be used wherever the bar graph or circle graph can be used. The pictogram is likely to be interesting but is misleading more often than other kinds of graphs.

14. The curved-line graph is used to picture gradual or continuous change, for instance, in temperature over a period of several days. Such a graph may be drawn from temperature readings taken at intervals of an hour or a half hour.

E. List of Readings for Teachers Only

E. R. Breslich, The Technique of Teaching Secondary School Mathematics, Chicago University of Chicago Press, 1930. P. 188

C. H. Butler and F. L. Wren, The Teaching of Secondary Mathematics, McGraw-Hill Book Company, New York, 1941

Periodicals

S. I. Altwerger, "The Art of Teaching Functioning Mathematics", The Mathematics Teacher, (1944) 37:314

W. M. Betz, "Whither Algebra", ibid (1936) 23:104-125

P. Stroup, "Thither Algebra", ibid (1930) 23:311-316

George H. Whiteaker, "Can We Teach for Meaning?", ibid (1945) 38:161

F. Unit Assignment

1. In order to understand the arithmetical and algebraic solutions involved in various accounts in our everyday living, it is often necessary for us to go a little further than we have gone in our prescribed courses in elementary grades. The average person uses mathematics in the buying of commodities such as food, clothing and medicine; in budgeting, allowances and family income. Much mathematics is used in figuring his savings, his investments, and his insurance.
2. If accounts, whether commercial or domestic, are to have any value for the average person it is necessary for the pupil to have some idea of the relationship existing between the cost and the selling price, the profit and the selling price.
3. Can you find the cost of an article if you are given the selling price and the profit, the cost if you are given the number of articles and the cost for each one?

(a) Find the cost of one dozen oranges at $4\frac{1}{2}¢$ each.
(b) What is the cost of 75 cans of milk at x cents per can?
(c) What is the cost of 8 pounds of sugar if it is 5¢ for 8 ounces?
(d) If an article cost the merchant $12.50 and he makes a profit of $1.25, what is the selling price?
(e) If S is the selling price, C the cost and G the gain, what is the formula connecting S, C, and G?
(f) Make a table of related values of S and C if G is $2.00 when C = $8. $10, $17, and $70.

4. In the preceding problem, we use the words gain and profit. Is there any difference? In the following exercises find the profit or the loss. Tell how you may determine whether it is a profit or a loss.

	Selling Price	Cost	Profit or Loss
A	$49.15	$45.00	?
B	5.90	5.50	?
C	.97	1.00	?
D	2.35	2.35	?

5. If +7 represents a profit of $7.00 then - 7 represents a loss of $7.00. What numbers then will represent the following:

A. A profit of $12.00
B. A loss of $3.00
C. A loss of $10.00
D. A profit of $2.50
E. A loss of $3/4
F. A profit of $1/20

6. In the preceding problems since profit and loss are considered opposite quantities they are represented by plus and minus signs. Can you name similar examples of opposites?

7. In these, as in other instances, we need to know how to combine these opposite quantities. Give the total effect produced by the following:

A. $5.00 gain followed by $3.00 loss.
B. Driving 12 miles north followed by 4 miles south.
C. 15° rise in temperature followed by 2° fall.
D. $17.00 loss followed by $5.00 loss.

8. In Algebra, we may think of certain numbers as opposites to positive numbers and use them in solving certain types of equations and problems. In this unit, we shall learn how to perform fundamental operations on both positive and negative numbers. In Arithmetic, we subtract any number from another number which is equal or greater, such as:

A.	654	Minuend
	444	Subtrahend
	210	Remainder

But subtraction like the following:

B.	3210	Minuend
	4444	Subtrahend
	????	Remainder

does not appear in Arithmetic.

Now notice in (A) above that as the minuend decreases, the remainder also decreases. Until the minuend and subtrahend are equal when we obtain 0 for our remainder. Note in (B) that our minuend is less than the subtrahend, therefore, our remainder must be smaller than 0.

Hence: The subtracting of a greater number from a smaller number is possible only by thinking of numbers smaller than 0.

A negative number is an invention which permits us to subtract B from A in A-B when B is larger than A. It is a number preceded by the - sign. Positive numbers have no sign at all, or sometimes a + sign. Thus we see that the sign (+) and minus or negative (-)

have two uses each. When they indicate addition or subtraction, as in solving problems, they are signs of operation, as when standing alone, they are called signs of quality. The absolute value of a number is the number of units it contains regardless of its sign. Thus the absolute value of either +10 or -10 is 10, since there are ten units in either number. Signed numbers may be illustrated by a diagram. Below is a scale in which the absolute value of the numbers to the right and left of zero is the same:

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10

But the points at the right of zero are considered positive and so are numbers +1, +2, +3, .., while those at the left are considered negative and so are numbered -1, -2, -3 .. Thus we see that every positive or negative number might be represented by some point on an extension of this line. Positive and negative numbers may be thought of as directed numbers.

As on the preceding scale, so on thermometers, a certain point is marked 0, and called zero. If we read in the paper some winter morning that the lowest temperature recorded on the preceding day was -10 degrees, what should we understand that statement to mean? By a similar method, how should we indicate 5 degrees above zero?

If distances north of a certain point are considered positive, then distances south of that point would be negative; if distances east are considered positive, distances west are negative.

Dates are reckoned in Christian countries from the birth of Christ. Dates since Christ (A.D.) may be considered positive, and those before Christ (B.C.) negative.

Numbers representing assets, that is, actual property owned, may be considered positive; and liabilities, that is, debts or obligations, which are the opposite of property owned, may be considered negative. From these illustrations, we can see that positive and negative numbers are opposite of each other in time, direction and character. They start in opposite directions from the point zero.

It is possible to perform the same fundamental operations on directed numbers as on positive numbers in arithmetic. Let us now study how to add, subtract, multiply and divide these opposite qualities as suggested by our study of profit and loss.

A. First let us consider addition by a scale similar to the one mentioned before. Let us draw a straight line of indefinite length and mark a certain point on it O. Then mark off equal divisions in both directions from O, numbering the divisions to the right with the positive integers, and those to the left with negative integers. To add +4 to +3, begin at +3 and count 4 spaces to the right, what is your answer? In the same way, we may use this scale to add +6 to -4. We first locate -4 on the scale and count to the right 6 units, which brings us to +2. This is the sum.

If we were to add two signed numbers as -4 and -2, we should begin at -4 and count 2 units to the left, giving us as the answer -6. These illustrations lead to the following:

Rule ___ To add two signed numbers by a scale, locate the first number on the scale. From it count as many units as there are in the second number, to the right if this second number is positive, and to the left if it is negative. The number reached is the sum. Using the scale, add the following:

(A)	(B)	(C)	(D)	(E)	(F)
+5	-7	+7	-6	-0	-9
-4	+10	-7	-4	+6	+9

B. In the beginning of this unit we saw how unlike quantities can be combined as profit and loss. Thus, if a merchant realizes a profit of $549 one week and suffers a loss of $86 the next week, we can express his total profit (or loss) by combining the two. If we set them down as sums and by studying these results we are able to state the following rule for the addition of signed numbers:

Rule ____ To add two numbers having like signs find the sum of their absolute values and prefix their common sign.

To add two numbers having unlike signs, find the difference between their absolute values and prefix the sign of the number having the greater absolute value.

9. Add each of the following, remembering that an unsigned number is positive:

(A)	(B)	(C)	(D)	(E)	(F)
-1	-3	-17	-14	-15	+15
+3	-6	+11	22	- 8	- 3

(G) 6 Dollars
5 Dollars

(H) -6 Dimes
+4 Dimes

*10. Supplementary material:

A. The following exercises will give you further practice in adding positive and negative numbers. (If further help is needed see (1:49-50)

(A) $16a + 3a - 5a$
(B) $10b + 6b - 3b$
(C) $\frac{x}{3} - \frac{2x}{7} - \frac{6x}{5}$
(D) $7x - 3 - 12x + 7 - 12$
(E) $3x^2 + 5 + 7x^2 - 8 - 4x^2$

B. Adding polynomals (1:51) Do five problems in (1:52)

11. Subtraction of signed numbers.--If you make a purchase of 68 cents and hand the clerk a dollar bill, he may hand you in succession 2 cents, 5 cents, and 25 cents, saying as he did so "70 cents 75 cents, a dollar." This method of making change is familiar to every boy and girl. The clerk has found the difference between 68 cents and a dollar by determining what must be added to 68 cents to make a dollar.

(A) What number added to 6 makes 8? Then $8 - 6 = ?$
(B) What number added to 0 makes 8? Then $8 - 0 = ?$
(C) What number added to -2 makes 8? Then $8 - (-2) = ?$
(D) What number added to -8 makes 8? Then $8 - (-8) = ?$
(E) What number added to 4 makes -3? Then $-3 - (4) = ?$

12. Just as in Arithmetic you found what number must be added to 6 to

* Throughout these units numbers in parenthesis following exercises will denote the reference and page. Thus (1:49-50) means the first reference pages 49 and 50.

9. [illegible] of the following, remembering that an assigned number is positive:

(A) -1	(B) -4	(C) -17	(D) $-\frac{7}{12}$	(E) -14	(F) -18
-3	-4	-11	$\frac{12}{12}$	-8	-5

(G) 8 dollars	(H) -4 dimes
5 dollars	-1 dimes

10. Supplementary material:

A. The following exercises will give you further practice in adding positive and negative numbers. The further help is needed see (1):[illegible].

(A) $14a + 3b - 5a$ (C) $7p - 3 - 18p + 7 - 12$
(B) $10b + 6a - 3b$ (D) $3a^2 + 7a^2 - 2 - 4a^2$
(C) $\frac{a}{b} - \frac{2x}{7} - \frac{6x}{5}$

B. Adding polynomials (1:61) do five problems in (1:[illegible])

11. Restriction of signed numbers.--[illegible] you want a purchase of [illegible] cents and have one class a dollar bill, so may want you in [illegible] 5 cents, 5 cents, and 25 cents, [illegible] so "70 cents 75 cents, a dollar." This method of adding [illegible] is possible to [illegible] and girl. [illegible] the difference between [illegible] cents and a dollar by determining that must be added to [illegible] to make a dollar.

(A) [illegible] 8 [illegible] Then [illegible]
(B) [illegible] 8 [illegible] Then [illegible]
(C) [illegible] 8 [illegible] Then [illegible]
(D) [illegible] 8 [illegible] Then [illegible]
(E) [illegible] Then [illegible]

12. [illegible] in [illegible] and number [illegible] to

* [illegible] Following exercises will [illegible] Then (1:[illegible]) [illegible] the first [illegible] pages [illegible].

give 9 and wrote $6+?=9$, here we use the process to find what number must be added to a given number to get another. This we call subtraction. The clerk in the store gives us our change by adding the amount of the purchase to the amount of change he is handing us.

A five-dollar bill is handed a clerk to pay for an article costing \$.63. Explain a simple method of making change.

There are many ways of reading such a problem.

(A) What is the result of subtracting 2 from 5?
(B) What is the difference between 5 and 2?
(C) How many units is it from 2 to 5?

With the use of minus sign and the negative numbers, subtraction now becomes the addition of a number with a minus sign. Thus to subtract 5 from 7 and to add -5 and +7 is the same operation; the result is +2 in both cases. The use of negative numbers makes it possible to subtract a larger number from a smaller number. For example, to subtract 7 from 5 simply add -7 and +5 and get -2.

Rule ___ To subtract a number, change the sign of the subtrahend and add the resulting number to the minuend. (The changing of the sign should be done mentally)

13. Using the above rule subtract the lower number from the upper:

(A)	(B)	(C)	(D)	(E)
+28	-11	+36	-8	+24
-10	+3	+9	-8	-17

14. Subtract the upper numbers from the lower in No. 13.

15. Multiplication of Signed numbers:

In Arithmetic we have been accustomed to regard multiplication as a short process of addition. For example, 4 times 5 is equiva-

lent to 5+5+5+5. . Similarly, in Algebra, we may regard multiplication by a positive number as a short process of addition, while multiplication by a negative number is a short process of subtraction. Thus:

(A) (+4) x (+3) = +3+3+3+3= +12
(B) (+4) x (-3) = +(-3) +(-3) +(-3) +(-3)= -12
(C) (-4) x (+3) = -(+3) -(+3) -(+3) -(+3)= -12
(D) (-4) x (-3) = -(-3) -(-3) -(-3) -(-3)= +12

What is the sign of the result if two positive numbers are multiplied? If two negative numbers? If one positive and one negative? What, then, is the product of -3 and +5? of 6 and -4? of -5 and -2? of -8 and 2? What is the value of (-2) (-6) if in Algebra the lack of a sign between two quantities means that they are to be multiplied? What is the value of 3(-11)? (-6) (+10) (-7)= ? 7.7 = ? (-2) 7 = ?

By inspection of the above results, we infer the following:

Rule____ When two numbers having like signs are multiplied, the sign of the product is + ; when two numbers having unlike signs are multiplied, the sign of the product is _____.

In connection with multiplication we must remember that xy means x times y, while 24 means 20 +4. This is one of the chief differences between the notation of algebra and that of Arithmetic.

When three or more numbers are multiplied, the multiplacation can be performed in any order. If there are no negative quantities or if their number is even, the sign of the product is +. If the number of negative quantities is odd, the sign of the product is negative.

Work the following:

1. 3 times 4; 3(-2); 5(-8); -3(6); -2(10)
2. (-4) (-8); (+3) (-6); (-7) (+5); (-8) (-3)
3. (-3) (-5) (-10); 3 (2/3) (-5/2); 4 (-6) (-2) (-1)
4. (-3) (-5); 2($-7\frac{1}{2}$); -6 ($\frac{1}{2}$); -4 (3) (-1)

*Work 1-25 (3:214-215)

16. Division of signed numbers:

Division is the opposite of multiplication. If the product of two numbers is divided by one of the numbers, the quotient is the other number. Thus, since 2 x 5 = 10; $\frac{10}{5} = 2$ or $\frac{10}{2} = 5$

(A) (+4) (+7) = +28, therefore, $\frac{+28}{+7} = ?$

(B) (+4) (-7) = -28, therefore, $\frac{-28}{-7} = ?$

(C) (-4) (+7) = -28, therefore, $\frac{-28}{+7} = ?$

(D) (-4) (-7) = +28, therefore, $\frac{+28}{-7} = ?$

From the above illustrations we infer the following:

Rule____ The quotient of two numbers having like signs is positive, while the quotient of two numbers having unlike signs is negative (division by zero excepted)

17. Perform the following operations:

1. 6 ÷ 3; -6 ÷ (-3); +6 ÷ (-3)
2. (-8 ÷ 2; 14÷ (-7); (-12) ÷ 4; -10 ÷ $2\frac{1}{2}$
3. $\frac{20}{-4}$; $\frac{-32}{8}$; $\frac{-50}{-10}$; $\frac{18}{2}$; $\frac{-49}{-7}$
4. 24 ÷ (-6) ; (-16) ÷ (-8) ; 25 ÷ (-5) ; (-18) ÷ (-9)
5. $\frac{0}{2}$; $\frac{0}{5}$; $\frac{-6}{-1}$; $\frac{-12}{-6}$; $\frac{28}{-7}$
6. 8 ÷ 4.5 ; (-6) (-5) ÷ 2 ; (-15) ÷ (-3) (-8)
   (-7) (-14) ÷ (-2). 3

7. What is the average of $5000 gain, $3000 loss, $2000 gain, $4,000 gain, $5,000 loss? (Answer $600 gain)

8. I have $100 in the bank after which I earn $200, spend 150, and borrow $300 from Mr. Jones. Mr. Williams owes me $250, Mr. Brown owes me $150. I owe Mr. Henry $50. How much am I worth?

* 9. Supplementary work
Work numbers 13-20 (3:204)

BUDGETING

If, in the process of buying and selling, we use positive and negative numbers to denote profit and loss, likewise in budgeting the ideas of oppositeness may also be used in expenses and income.

According to Cowan, "A budget is an estimate of probable income and expenditures."[1/] (2:304) In this definition our income will be considered the positive quantity while the expenditures will be considered the negative quantity.

In order to understand clearly the principles of budgeting, it is necessary to have some means of relationship of one item to another. Here we may employ means of graphing.

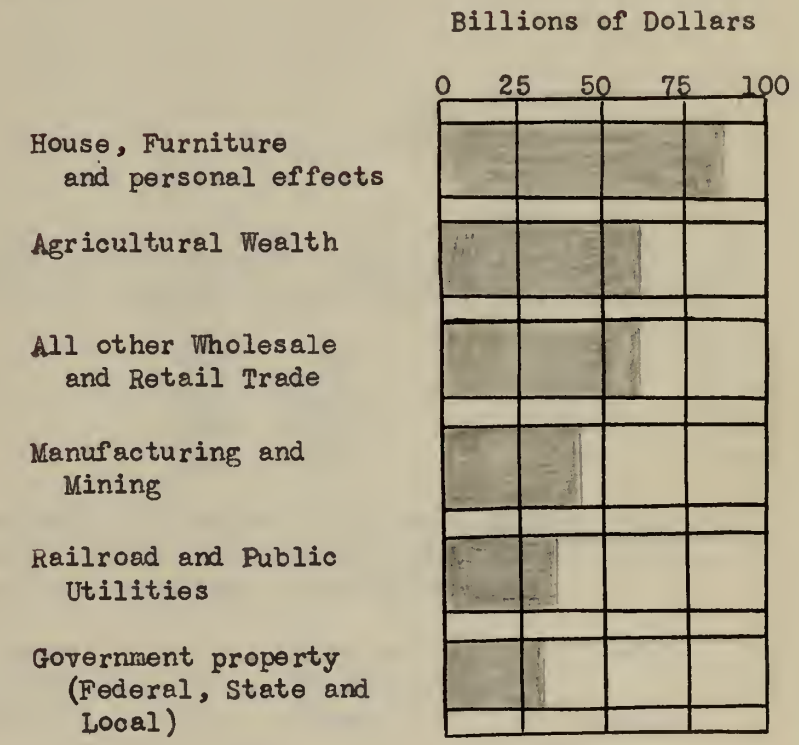
You have very likely had some experience with graphs before, possibly in your study of arithmetic as well as in your reading of magazines and newspapers. Would you like to know at this time how much you have remembered about graphs from your previous study and observation of them? Here is a special test for the purpose of letting you see where you stand as you begin work on this topic.

1/ A. L. Cowan, Consumer Mathematics, P. 304

The purpose of this test is to show you and your teacher what knowledge you already have of graphs and thus afford a basis from which progress can be measured. So, if you find that you have not done very well on the pretest--do not feel discouraged; in the study of this unit you will be given an opportunity to increase your knowledge. If you have answered all of the questions correctly, you should be able to complete this part of the unit in a very short time.

PRETEST

Directions: Fill in the blanks in the questions below by interpreting (reading) the following graph.



Graph I

Uncle Sam's Wealth: Houses, Farms, Stores, Factories, Mines, Railroads, Public Utilities, Government Property

1. What kind of graph is graph I? _____

2. What items account for the greatest part of Uncle Sam's wealth? _____

3. What two items are equal? _____ and _____

4. Manufacturing and mining are worth _____ billions of dollars.

5. Each space represents _____ billions of dollars.

6. A boy spent his dollar as follows: 20¢ for an ice cream soda, 25¢ for a baseball game, 30¢ for a movie, 15¢ for candy, and put the rest in his bank. Represent these facts by a circle graph. Label the graph properly.

7. Using your graph paper construct a broken-line graph of Jack's earnings for a year. Label it properly.

Jan.	$3.00	May	$3.50	Sept.	$6.00
Feb.	2.00	June	4.50	Oct.	5.00
Mar.	2.50	July	25.00	Nov.	4.00
Apr.	4.00	Aug.	20.00	Dec.	3.50

8. Construct a rectangle 24 squares long and 2 squares wide and in it show the following relations applying to a pupil in junior high school. Label the graph properly.

School	6 hours	Play and work	4 hours
Sleep	9 hours	Other uses of time	4 hours
Study	1 hour		

9. On a winter day the hourly temperature readings from noon until midnight were as shown in the following table. Make a curved-line graph to show these facts. Label it properly.

Hour	12	1	2	3	4	5	6	7	8	9	10	11	12
Temperature	22°	24°	25°	26°	21°	18°	19°	15°	15°	10°	8°	5°	2°

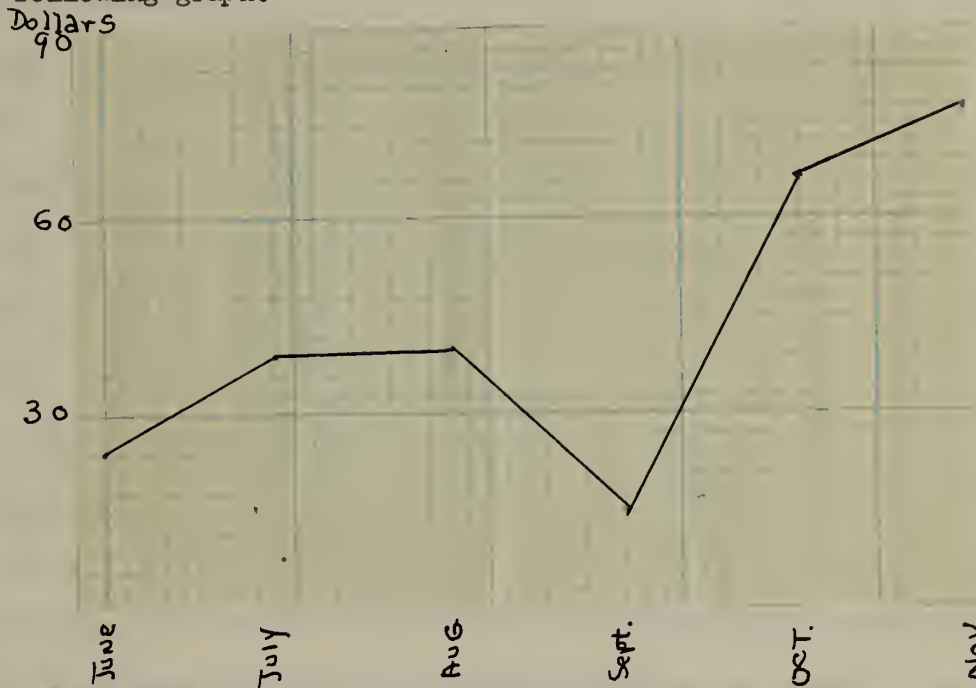
** Supplementary material:

During your study of this topic, try to find examples of graphs used in newspaper and periodicals. These will show you the practical value of being able to make and interpret graphs.

PRETEST

Part II

Directions: Fill in the blanks in the questions below by interpreting the following graph.



1. What kind of graph is the above graph? _____

2. During which month was Raymond's account the smallest? _____

3. During which month was his account the largest? _____

4. How much money did Raymond have in his account in June? _____

5. Each space represents _____ dollars.

Throughout this unit you will find groups of problems for arithmetic fundamentals and practical and social mathematics. The study of these sections will give you an opportunity to become familiar with a large number of problems of the type which you are most likely to use in everyday life.

A graph is a picture showing the relative size or importance of two or more things. The purpose of a graph is to make it possible to see very

easily and quickly how these things compare. In the same way that a movie tells a story more vividly than a book, so a graph tells a story more vividly than a column of numbers.

The Picture Graph

Read: pages 1 and 2 of General Mathematics[1] by Edgerton and Carpenter.

A picture graph tells its story so that practically no reading is necessary. While picture graphs are very interesting and effective in many cases, they are not used so extensively as some of the other kinds of graphs.

The Bar Graph

Study: pages 3 and 4 of your text. Then study the material which follows in this unit. The bar graph is one of the most common types of graph; you should, therefore, be careful to master this material. A bar graph may be constructed either perpendicularly or horizontally, as the two graphs on page three of the text illustrate. First we shall learn how to interpret or read bar graphs. The first graph on page three shows the marks obtained in a certain test in mathematics. Two questions are asked in connection with this graph:

1. How many took the test?
   The graph consists of four bars labeled E,G,F and P. bar E extends up 5 squares, showing that 5 pupils received a grade of E. Bar G extends up 14 squares, showing that 14 pupils received a grade of G. Bar F extends up 18 squares showing that 18 pupils received a grade of F. Bar P extends up 3 squares, showing that 3 pupils received a grade of P.

To find how many took the test add 5, 14, 18, and 3, which gives 40

[1] This will be referred to as our textbook for the topic of graphs.

pupils who took the test.

2. What per cent failed?
Bar P shows that 3 pupils failed. To find what percent this is of the total number of pupils, divide 3 by 40, which gives 3/40 or 7½% (3 divided by 40 equals .075).

When working with a large number, you will find that a bar graph is not absolutely accurate; but with it you can make a close approximation. Try to figure out how much was spent for each of the five items listed in the horizontal bar graph by studying the graph. Then read the figures given and see how nearly right you are.
At the bottom of page 3 and the top of page 4 of the text you will find an explanation of how to draw bar graphs and suggestions for making such graphs. Study this material carefully.

To make clear the instructions, let us work together number 5 of the text. Count off 18 squares across the top of your graph paper, the longest bar must represent 357 ships; therefore let each square represent 20 ships.

The United States had 357 ships; hence the bar for this navy will be slightly over 17 4/5 squares long. In the same way the bar for Great Britain is found to be 13 4/5 squares long; for Japan's, 11 squares long; for France, 9 4/5 squares long; for Italy, 8½ (slightly over) squares long.

In choosing the position of your graph, locate it so that the margins will be as nearly equal as possible. Neatness is very important in making graphs.

Work: Exercise 2, page 4; exercises 3 and 6, page 5; and exercise 7, page 6.

The Double - (Dual-) Bar Graph

A double-bar graph is a special type of bar graph. Graphs of this type are constructed in the same way as the bar graphs we have been discussing, except that there are usually only two bars and these are placed against one another.

Study: the section on Double-bar Graph, page 6 of your textbook, and answer the questions given there.

Work: Exercise 2, page 6.

The Rectangular Distribution Graph

Read: page 7 and 8 of your textbook to see how this graph is constructed. We shall work problem 4 step by step so as to make sure that the explanation is clear.

If we make bar 4 3/8 inches long, 1/8 inch will represent one period (1/8 x 35 = 3/8). Since 5 periods are given to English, the section for English will be 5/8 inches long. Latin, 5 periods, will have a space of 5/8 inches. History 4 periods, will take 4/8 or $\frac{1}{2}$ inch. Chemistry, 7 periods, will occupy 7/8 inch. Physical training, 2 periods, will take 2/9 or $\frac{1}{4}$ inch. Study periods, 12 periods (35 - (5 + 5 + 4 + 7 + 2) = 12) will take 2/8 or $1\frac{1}{2}$ inches.

The Circle Graph

The circle graph is sometimes called a "pie" graph. In order to construct a circle graph you must have a compass or some instrument with which

to draw the circle, and a protractor which is used to measure the angles. You will find the protractor in the pocket inside the cover of your textbook.

Study: pages 8 and 9 of your textbook.

You must know that a circle contains 360°. Study the example on page 9 of your textbook. To find the fractional part of the circle which represents each different item, divide each number by the total and change the fraction to a percent. In the example on page nine the total annual salary is $2000. For rent, $300 is spent which is $\frac{300}{2000}$ or 15% of his entire salary. Fifteen percent of 360° equals 54°, the part of the circle which will represent the portion of salary spent for rent.

Illustration: In drawing a circle graph to illustrate the method, use exercise four page ten of your textbook.

First it will be necessary to determine how many pupils are enrolled in high school. To do this we add 1780+ 975 + 595 + 425, which gives 3775. Next we need to find how many degrees each class will represent in the circle.

$\frac{1780}{3775}$ = .47; 47% of the pupils were Freshmen.

47% of 360° = 169° for Freshmen

$\frac{975}{3775}$ = .26; 26% of the pupils were Sophomores

26% of 360° = 94° for Sophomores

$\frac{595}{3775}$ = .16; 16% of the pupils were Juniors.

16% of 360° = 57° for Juniors.

$\frac{425}{3775}$ = .11; 11% of the pupils were Seniors.

11% of 360^{0}=$40^{0}$for Seniors.

By working through the actual calculation you will find that the percents and degrees did not come out quite even, but to make the work more simple we have changed fractions which were over one half a whole and dropped those which were less than one half.

If we have made no mistake in our calculations, the degrees should total 360^{0}.

169^{0} 94^{0} 57^{0} 40^{0} 360^{0}

The next step is to draw the circle. For this we use our compass and any convenient radius. Then we draw a line from the center of the circle to any point on the edge of the circumference of the circle. Next we place the protractor on the circle so that the arrow on the base of the protractor falls exactly on the center of the circle and the line marked zero falls along the line drawn from the center of the circle. We read around the protractor to 169 and put a little mark there. Then we place the ruler edge of the protractor so as to join this point and the center of the circle and draw a line from the center of the circle until it intersects the edge of the circle. We write in this section "Freshmen, 47%." Again we place the protractor on the circle so that the arrow falls on the center of the circle, and label this section "Sophomores, 26%." We continue in this manner until all of the items have been represented and the graph is complete.

Work: Exercises 1, 2, and 3, pages 9 and 10 of the textbook.

Frequency Distribution Graphs

The construction of the frequency distribution graph is like that of the double-bar graph, except that there are several bars rather thanjjust two.

Study: pages 10 and 11 of your text book, especially the graph which has been worked out on page 11. Answer in your own mind the questions given there.

The Broken Line Graph

Study: pages 11 to 13 of your text book to learn how to construct a broken-line graph.

In drawing a graph of this type the two base lines or axes should be placed at the left and bottom, as shown in figure on page 12 of the text book. The point where these two lines come together is called the origin. The horizontal axis is usually used to represent the time such as days, weeks, months and years. The vertical axis is used to represent the quantities. The most important marks are the individual points on the broken-line graph. The lines joining these points merely show how much difference there is between the points and make the graph stand out more clearly.

On page 13 of your textbook there are six questions given under interpretation. You should be able to answer these questions.

You will not be asked to draw any broken-line graphs until you have studied curved-line graphs. The construction of these two types of graphs is very similar.

The Curved-Line Graph

The curved line graph is constructed exactly as the broken-line graph

is, except that a curved line rather than a straight line is to connect the points.

Study: The explanation, questions and graphs given on pages 13, 14 and 15 of your textbook.

When you are representing things that change gradually, such as population or temperature, use a curved-line graph. When you are representing things that change abruptly, such as crops from year to year, use a broken-line graph.

Draw: the graph required in Exercise 1 and 3, page 16 of the textbook, and Exercise 4 and 7, page 17, using curved or broken lines, whichever you think is better suited to the problem.

Comparison by Graph

It is often desirable to draw two graphs on the same set of axes. In this way the relation between two changing factors can readily be seen. Each graph is constructed in exactly the same manner as those in the preceding section except that they are both placed on the same set of axes.

Study: the material on page 18 of your textbook and work Exercises 2 and 3 on page 19.

Suggestions for Using Graphs

You have studied suggestions for the use of each kind of graph as you learned how to make that type of graph, but review them again so as to make sure you have them well in mind.

Study: page 20 of your textbook.

TEST ON GRAPHING

Make the most suitable graph for each of the following:

I. Mr. Carrol's dollar was used as follows:

Living expenses	50%	Charity	10%
Education	8%	Savings	20%
Recreation	12%		

II. The scores in a mathematics test.

Number	Score	Number	Score
2	0-9	5	50-59
2	10-19	6	60-69
3	20-29	7	70-79
1	30-39	2	80-89
2	40-49	3	90-99

III. The amount of electric current used by a family for 6 months in Kilowatt hours was: January, 80; February, 90; March, 98; April, 70; May, 70; June, 65.

IV. How a city spends its dollar.

Education	24 cents	Interest	28 cents
Police	14 cents	Sanitation	9 cents
Fire Dept.	6 cents	Hospitals	4 cents
County Expense	3 cents	Miscellaneous	12 cents

Paying Bills

In the keeping of accounts there is great need of an adequate knowledge of careful buying and of budgeting our income. We have seen how graphs show clearly the relationship of one item on the budget to the other items. Among the items on the family's budget, household expenses rank high. This fact is vividly illustrated in the following account as a Father looks over his mail. "The first of the month again," said Mr. Ted Henry as he looked over his monthly bills. "I wonder whether the pay check will cover them. Let's see, here are those charge accounts, a small bill from the dentist, and our friends, the electric light bill, the gas bill, the telephone bill and the water bill." He added all the bills and consulted his budget. Mr. Henry would have found himself in an awful predicament, if he did not have a budget.

Two of the bills that Mr. Henry had before him were the electric light bill and the gas bill. These are two of the most common bills in every household.

Do you know the principles involved in the use of electricity? of gas?

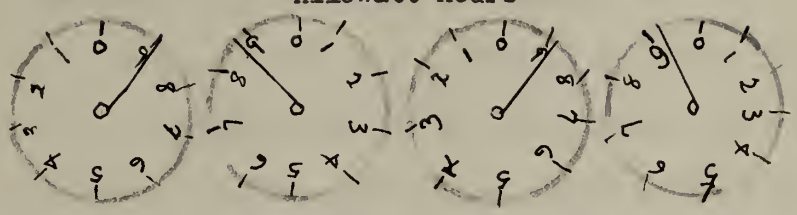
Electricity, like many commodities that are sold, must be measured. The unit of measurement is the watt hour. A light bulb marked "40W" burns 40 watt hours of electricity in one hour. Electrical appliances such as the toaster, the sweeper, the ironer, and the washing machine, all use a certain number of watt hours of electricity. A kilowatt hour is one thousand watt hours.

Household Accounts

Electricity

Electricity is measured in Kilowatt hours.

Kilowatt Hours



Notice that two are read clockwise, and two counter-clockwise. Read the dials from left to right. A hand between two numbers is read as the preceding number. Thus these dials read: 8889.

I. Secure as many receipted electric bills as you can and construct dial diagrams to represent the different amounts.

II. Take meter readings at home or elsewhere for three consecutive days, and compute the charges for the three days on the basis of local rates.

III. What are the rates charged for electricity in your city?

The rates charged vary in different localities, but the following schedule is typical:

First	50	KWH	per	month,	4	cents	per KWH
Next	50	"	"	"	$3\frac{1}{2}$	"	" "
Next	100	"	"	"	2 3/4	"	" "
Over	100	"	"	"	$1\frac{1}{2}$	"	" "

Minimum charge per month 60 cents.

IV. Mr. Simpkins electric light bill for one month is $3.68. The bill states that he used 98 KWH. Verify this from the table.

V. Find the cost of burning a cluster of fine 60-watt bulbs, which burns on the average 3 hours, 15 minutes per day for a month of 30 days.

Method:

5 x 60W = 300W
$3\frac{1}{2}$ x 300 WH = 975 WH per day
30 x 975WH = 29250WH per month
29250 ÷ 1000 = 29.25 KWH per month

At 4 cents per KWH, the cost is 29.25 x 0.04 = $1.17

VI. In problem 5, it was necessary to divide by 1000. This should be done by moving the decimal point three places to the left. How would you divide by 10? by 100?

VII. Find by short cut the answers to the following examples:

(a) 92 ÷ 10 = ?
(b) 92826 ÷ 100 = ?
(e) 4800 ÷ 100 = ?
(g) 2.14 ÷ 10 = ?
(b) 45 ÷ 100 = ?
(d) .9 ÷ 10 = ?
(f) 3620 ÷ 10 = ?
(h) .52 ÷ 1000 = ?

Gas Bills

1. What are the rates charged for gas in your city?
2. What unit of measurement is used for gas?
3. All gas meters are much alike. The dial at the top is simply a test dial; those at the bottom measure the gas used. The figures denote tenths of the numbers over the dials and so two ciphers must be added to the reading of the right-hand dial. The difference between any two readings measures the amount of gas used for that period (4:88-89).

4. Study the problems on pages 86 through 91 in this fourth reference and be able to answer the questions.

*5. Collect receipted gas bills and be able to draw dials to represent them.

*6. Repeat problem 5 for water bills instead of gas bills.

Test on Household Accounts

On a sheet of paper complete the following statements by supplying the missing words. Write answers only. Number the answers to correspond with the numbers of the sentences.

1. 60 W printed on an electric light bulb means .....
2. 1 KWH=..... Watt hours
3. 400 Watts = ... Kilowatts
4. 1 hour = ... minutes
5. 1 cubic foot = ... gallons

In the following sentences, choose the correct word, group of words, or figure on the same sheet of paper. Write your answers in a column, numbering the answers to correspond with the numbers of the sentences.

6. The cost of electricity per KWH (increases, decreases, remains the same) as the number of KWH consumed increases.

7. Of the two types of electrical appliances, the (motor type - heating type) usually consumes more current in a given time.

8. The cost per M of gas (increases, decreases, remains the same) as the amount of gas consumed increases.

Solve the Following Problems

1. Write $1\frac{1}{4}$ cents, using the dollar sign.

2. Write 3.5 cents, using the dollar sign.

3. (a) $842 \div 10 = ?$ (b) $8.42 \div 100 = ?$ (c) $10420 \div 1000 = ?$
   (d) State a rule which gives a short method for dividing by 10; a rule for dividing by 100; a rule for dividing by 1000.

4. Multiply (a) 45 by 10; (b) 165 by 100; (c) 8.452 by 1000;
   (d) State a rule for multiplying by 10; a rule for multiplying by 100; a rule for multiplying by 1000.

5. Change to decimals:
(a) $\frac{1}{10}$; (b) $\frac{1}{100}$; (c) $\frac{1}{1000}$ (d) $\frac{5}{10000}$ (e) $\frac{4}{5}$ (f) 1 3/4%

6. Find the cost of operating 6 40-watt bulbs 4 hours, 30 minutes per day for 30 days at 2 3/4 cents per KWH.

7. Draw four dials and show the positions of the hands if the reading is 7653 watt hours.

8. If a gas burner consumes 2 cubic feet of gas in 1 minute, how much does it use in 3 hours?

G. Commentary.— In this unit on accounts, though it does not cover all the general field of the subject, it treats enough of the special application to give a working knowledge of their fundaments that a pupil may continue his studies if such a unit were not studied.

Practical problems were used because certain phases of mathematics are needed by the average person every day of his life. Each day the consumer uses not only the fundamentals of mathematics, but also the concepts and principles of percentage, intelligent buying, insurance and savings programs. As a citizen he should be able to read gas and electric meters as well as to understand and use the mathematics of a contract in installment buying.

From this unit on accounts it is hoped that the pupil will become more conscious of the need of mathematics in his daily occupations and avocations. The investor computing the interest due him, the father planning an annuity for his son's future education, the person preparing for civil service examination, the harassed taxpayer calculating his exemptions--all have need of functioning mathematics.

In an article called "The Defeat of the Schools" in The Atlantic for March, 1939, Professor James L. Mursel for Teachers Colleges, Columbia University, states the shortcomings of our present-day program of secondary mathematics to provide functional competency. Starting with mathematics he names it the most precise and powerful instrument for abstract thinking but charges that if one has not learned to think in its special language one has not learned to think at all.

This unit is intended, in part, as an answer to the editorial on the

above report entitled, "What are we going to do about it?" 1/ in which the editor offers a challenge to present day teachers.

In this unit there were changes in the content and in the arrangement but as Reeve points out, "These alone cannot produce any great improvement. The difficulty is deeper and includes the teaching side of the problem as well." 2/

The problem of accounts is a major concern to everyone. In the outset the problems of selling price, cost, profit and loss are taught for the principles concerned and their effects upon the consumer. The section on careful or intelligent buying is introduced for the sole purpose of making the pupil conscious of wise choice in spending. The problems of budget, gas and electricity will be discussed, as near as possible, in regard to what the pupil has already come in contact with or how these social phases of mathematics affect him. The phase of accounts usually considered in most texts will not be taken up here but will be gotten as supplementary work if the class shows a need for it. In this section the neglected, important phases of this work will be emphasized most.

1/ W. D. Reeve, "What Are We Going To Do About It?" The Mathematics Teacher 32 (1939) p. 181

2/ W. D. Reeve, "Modern Curriculum Problems In Teaching of Mathematics in Secondary Schools" ibid. p. 128

CHAPTER IV

UNIT ORGANIZATION OF THE TOPIC, INSURANCE

The Time:

(Tentative time allotment, three weeks, five times a week)

A. General Statement of the Unit.-- The term "insurance" is here used blanket fashion to cover those forms of ordinary insurance with which a child may reasonably be expected to come in contact, such as life, fire, automobile and accident insurance. It is hoped that through various applications there will grow a clearer understanding of the terms used. The main points of insurance will be told, as near as possible, in story form beginning with its history and stressing as many phases as is possible in this unit ending with the most modern form of insurance-- Veteran insurance.

Objectives: This unit was constructed with the following objectives in sight:

1. To give the pupil a basis and a background for appreciation and understanding of insurance in its many forms.
2. To give the pupil sufficient mathematical ability to understand and compute the finances of insurance.
3. To give the pupil a more accurate and ready working knowledge of the four fundamentals operations.

B. Detailed Description of the Unit.-- This unit will be developed in the following manner:

The introduction gives the stage of a young veteran just returning to civilian life who has a sum of money to invest. He has a wife and young son, an automobile and a home. He has never felt the need of insurance before but after much persuasion his wife finally gets his consent to visit several insurance agencies.

Jim Brown's visits here are combined with algebraic application as his wife, Thelma, is an elementary algebra teacher and is interested in making a study of just how much mathematics, algebra in particular, is used in this phase of our social life.

The first stop takes him to a life insurance company. Here Jim gains an insight into the history of insurance. He then learns the pertinent facts regarding ordinary, endowment and limited payment life.

The next stop takes Jim to another company where, in addition to the things he has heard before, he learns about short term policies, annuities, and accident and health insurance.

Thelma then suggested that Jim also investigate other insurances on the automobile, the dwelling house, the garage and furniture.

Jim's interesting visits end at the Veteran Administration Office where he investigates some of the things that he heard mentioned while in service.

C. Delimitation of the Unit

1. An endowment policy is a contract to pay a certain amount to the insured at the end of a certain period or on his death.

2. An ordinary life policy is written for life or 96 years.

3. The chief purpose of life insurance is protection.

4. The owner of an endowment policy pays the premiums for a limited number of years.

5. An endowment insurance gives protection and investment.

6. An endowment insurance costs more than life insurance.

7. A limited-payment life policy is for a specified number of years.

8. The limited-payment life is different from the endowment in that the money is not paid until the death of the insured.

9. A term-policy runs for a certain number of years and then expires.

10. In the term-policy the policy continues only while the policy is in force.

11. The term policy provides greatest financial protection for a certain number of years at the smallest possible cost.

12. A person may purchase a term life insurance policy to protect another person from whom he has borrowed money.

13. Costs of insurance may be illustrated on a graph.

14. Fire insurance is greater on a frame house than on a brick dwelling.

15. Fire insurance is more expensive on a garage than on a house.

16. Health insurance provides weekly indemnity for illness, double indemnity for hospitalization and stipulated indemnity for various operations.

17. An automobile owner can insure himself against liability, fire, theft, property damage and collision.

18. The cheapest insurance for a veteran is the National Service life insurance.

D. List of Readings for Teachers Only

Books

E. R. Breslich, The Technique of Teaching Secondary School Mathematics, Chicago University of Chicago Press, 1930. P. 188

C. H. Butler and F. L. Wren, The Teaching of Secondary Mathematics, McGraw-Hill Book Company, New York, 1941

B. D. Cole, Autobiography of a Successful Insurance Agency, The Rough Notes Company, Indianapolis, 1927. P. 203.

Sylvester C. Dunham, The Business of Insurance, The Ronald Press Co., New York, 1921. P. 595

Julia E. Johnson, Social Insurance, H. H. Wilson Co., New York 1922. P. 309

Periodicals

S. I. Altwerger, "The Art of Teaching Functioning Mathematics", The Mathematics Teacher, (1944). 37:314

W. M. Betz, "Whither Algebra", ibid (1936) 23:104-125

P. Stroup, "Thither Algebra", ibid (1940) 23:311-316

George H. Whiteaker, "Can We Teach for Meaning?", ibid (1945) 38:161

E. Unit Assignment

General Study and Activity Guide

1. Introduction: A class discussion about the many forms of insurance and the amount of mathematics needed to really understand and appreciate insurance in its many forms. Jim Brown has just returned from overseas and has a lump sum of money to invest. What are the safest ways of investing this for him, his wife, and son. They have among their possesions an automobile and a frame dwelling house. The discussion is centered around such questions as:

Boston University
School of Education
Library

(a) What are the safest investments today? Will it be wise for him to spend all of his cash on insurance? What form of insurance will be best for him, his wife, his son, his automobile, and his home? (This discussion will serve as a pretest ascertaining how much the pupils know about insurance). Other points to be brought out by discussions are: (a) The type of work that Jim is going to do, (b) What the standard of living for this young couple is or has been, (c) The employment of his wife, Thelma and her income bracket, and (d) The importance of knowing the son's approximate age. Discussion closes with a consideration of the following question: What part does mathematics play in all these calculations and what can Mrs. Brown get out of Mr. Brown's investigations to teach to her class in elementary algebra?

2. Let us visit the Brown's home for a few afternoons and work the problems raised in his visits.1/

As an introduction to the idea of insurance, the company reviewed the highlights of the History of Insurance for Jim. Historians seem to agree that marine insurance was the first form in which any insurance appeared. They do not seem to agree just where it did start. Some say the Greeks were the first. Others say the Italians. While still others believe it to be the Spaniards.

Fire insurance may be traced in its history back to the era before Christ. Provisions were made for loss by fire in different communes. A

1/ The remaining sections of this Study and Activity Guide will be reproduced in full for the pupils.

fund was established from contributions from each member of the commune. If members were too poor to pay their share, the other members paid it. In case of fire, after the magistrate determined if the fire was accidental, money was provided from the fund to repair or rebuild. These communes still exist in China. This type of insurance seems to have existed for hundreds of years.

The psychology of insurance has changed with the times. In the early days, persons considered it a privilege to obtain insurance. Today, one is deluged with mail soliciting insurance.

In the field of accident insurance, England was the pioneer. About 94 years ago, The Railway Passengers Assurance Company started issuing insurance tickets to the travelers. Many companies followed this pioneer in accident insurance.

In the United States, health policies came before accident policies. As early as 1850, there are records of companies writing such policies. An American, travelling in England, happened to buy one of the insurance tickets. Being interested, he visited the head of the insurance company obtaining valuable information. Upon his return home, he formed "The Travelers Insurance Company." In the history of life insurance, it appears that annuities, now being so extensively advertised, were probably the earliest form of life insurance. In Rome, in 230, A. D., annuities were sold on a totally unscientific basis. At the present time, the life insurance company seems to be able to offer annuities of attractive value with the necessary security. The earliest life insurance company was started in London, in 1699. The future history of life insurance seems to

be favorable both for the company and the policy owner.

The history of insurance is indeed an intersting one. The change from ordinary life insurance to endowment at age 85 is but one innovation. The tendency toward the development of annuities is probably the greatest current change. As economic conditions change, the policy of insurance companies will change to meet the new conditions, and new history will be in the making.1/

At the end of this phase of the work, Jim was much more interested than in the outset, so he asked the agent to go into the ordinary life policy.

Ordinary Life Policy

This policy is written for life or 96 years and the owner must pay premiums throughout his life. At death, the insurance company pays the face of the policy to the beneficiary. Its chief purpose is protection and is used to a great extent by people of average means with dependents. The premiums may be paid weekly, monthly, quarterly, semi-annually or annually. Next to term insurance it costs the least.

Endowments

An endowment policy is a contract to pay a certain amount to the insured at the end of a certain period or on his death if that occurs before the end of the specified period. The owner of an endowment policy pays the premiums for a limited number of years. If he dies within the period, the amount of the policy is paid to his beneficiary. An endowment policy

1/ G. B. Gallagher, "Insurance an Informational Unit for Junior High School Mathematics", Unpublished Masters Thesis, B. U. School of Ed.1938 P. 13.

gives protection and is also an investment. It is bought by a person to assure himself of an income in old age or to give his children an educa - tion. This kind of insurance costs more than other forms of insurance since the money actually must be paid by the insurance company at a stated time, whether or not the insured person is still living. It may be better for a person to carry another form of insurance at a lower annual cost and save the difference through a savings bank account.

Limited-Payment Life Policy

This is a contract to pay a specified sum on the death of the insured, the premium being payable for a specified number of years, or during the life of the insured if his death occurs before the end of the specified period.

This is a life policy, but the holder pays premiums for a limited number of years only. It is purchased by one who wishes to pay all the costs of insurance while he is young and have his old age free from the burden of payments. The insurance company does not pay the amount of the policy until the insured person dies. It is different from endowment insurance in that the insurer cannot collect the insurance at the expira- tion of the period during which premiums are to be paid. For this reason, this form of insurance costs less than endowment insurance.

Problems on Cost of Life Insurance

The following table shows the annual premiums on $1,000 for several kinds of policies taken out at different ages:

Age	Ordinary Life	20 Year Endowment	20 Year Limited Payment
20	$18.01	$47.54	$27.78
25	20.14	48.03	30.07
30	32.85	48.71	32.83
35	26.35	49.75	36.17
40	39.94	50.39	40.35

1. What is the yearly premium on a 20-year endowment policy for $5,000 taken out at the ages of 20, 30, 40?

2. What is the agent's commission in each case if it is 20% of the first premium?

At the end of the day, we find our Mr. Brown discussing the many forms of life insurance. The preceding problems grew out of their discussion. Mrs. Brown also included in her plans the following problems. Let us suppose that we are in her class and are writing out the solutions to the following:

Insurance-Making a Line Graph

Make a graph for the following showing the annual premium per $1,000 for three kinds of life insurance. On the same paper, using the same reference axes, make a graph of each and tell what the graph shows.

Age	Ordinary Life	20-Payment Life	20-Year Endowment
20	$13.48	$31.22	$40.85
25	15.10	32.54	41.10
30	17.19	34.52	41.39
35	20.11	37.58	42.33

Define the following:

1. Beneficiary
2. Premium
3. Policy
4. An annuity

Cumulative Review:

(a) Mrs. Mahoney's husband died leaving her $10,000 worth of insurance. Upon the application of Mrs. Mahoney, the life insurance company agreed to pay her the interest on this amount at approximately 4½ percent.interest as long as she lived, and, at her death, to pay the $10,000 to her daughter. What was her annual income?

(b) If she wished to receive her income monthly, the company agreed to pay her monthly but discounted each payment at 6 percent interest for the number of months remaining in that year. What would Mrs. Mahoney's payment be at the end of each of the months following the first year of her husband's death?

Hint: (For 1st month) $450 ÷ 12 = $37.50
This amount is discounted at the rate of 6 percent for 11 months.

6 percent of $37.50 = 2.25

$\frac{11}{12}$ of $2.25 = $2.06

$37.50 - $2.06 = $35.42 (Payment the 1st month)

The second month $\frac{10}{12}$ of $2.25 would be subtracted and so on.

Accident and Health Insurance

Accident and health policies are experiencing policies. "In probably no other branch of the faculty of imagination played more important parts than in the development of the accident policy."[1] However, based on statistics being constantly complied from history experience, both accident

1/ S. C. Dunham, The Business of Insurance, p. 21, The Ronald Press, New York, 1912.

and health policies are being reduced to a scientific basis. Accident and illness prevention are also playing a large part in the field of insurance.

While accident insurances are issued for a period of time, the health insurance provides weekly indemnities for illness, double indemnity for various operations.

Professional men, particularly doctors and dentists, whose income warrants it, usually take advantage of such insurance.

An ordinary accident policy excepts accidents due to a variety of causes, such as extra hazardous occupations or carelessness.

The problems that Jim studied are those given in (4:284-295)

Exercise:

1. Write out the solutions for these problems for the Browns.

Term Policy

A term policy is the one which is to run for a certain number of years and then cease to be in force. The payments continue during the time the policy is in force. If the insured lives beyond the term of the policy, he is no longer protected. It provides greatest financial protection for a certain number of years at the smallest possible cost. A person may purchase a term life insurance policy to protect another person from whom he has borrowed money. He will carry this policy until his debt will have been paid.

Problem

Jim Brown was interested in knowing just what was the amount of protection afforded by the many kinds of life insurance policies. The agency

demonstrated by using five circles as shown in (2:124), showing the following percentages.

Kind of Policy	Amount of Protection
Short term (All protection)	100%
Ordinary Life	65%
20-Payment Life	42%
20-Year Endowment	22%
Single Life Annuity	0%

1. Prepare these five circles for Jim to take home for comparison, using the facts you know about. Draw the circular graph.

At the end of the second day, the Browns composed the following:

Review Study Questions on Life Insurance

1. What is meant by short-term insurance?
2. What is the contract entered into with an insurance company called?
3. What does the name "term insurance" imply?
4. What is the object of term insurance?
5. What is the purpose of Limited-payment Life Insurance?
6. What is the difference between a Limited-payment Life Insurance and Endowment Insurance?
7. Name one good use of a Short-Term Endowment Policy?
8. As compared with a Limited-payment Life and an Endowment Policy, is the premium rate on an ordinary Life policy lower or higher?
9. Which type of insurance is best for one whose earnings are expected to cease at a definite time?
10. Is a good test for any policy to ask, "What do I want this policy to do for me?"
11. What is an Annuity Policy?
12. In what way is an annuity a 100% investment?
13. What is the cheapest form of insurance for life?

14. What happens if insurance premiums are not paid?

15. Are life insurance rates for young men and women lower than for older persons?

The Browns decided to consider the automobile insurance next. So the third day finds our character in the office of the automobile insurance company. The visit was written up thus:

Automobile Insurance

There are five things against which an automobile owner can insure himself; liability, fire, theft, property damage, and collision.

Liability Insurance protects him against a limited amount of damage in case he injures some one.

Fire Insurance protects him against damage to his automobile by fire.

Theft Insurance protects him against loss by theft.

Property Damage protects him in case his automobile damages any property such as another automobile or a plate glass window, and so on.

Collision Insurance protects him in case his automobile is damaged. This is of special benefit to one who has to leave his car standing in the street much of the time. For example, a doctor parks his car during his calls. He runs daily risk of damage to his car by careless drivers.

Practice in Buying Automobile Insurance

1. In Miami, the cost of fire insurance on a used automobile was $1.17 per year. What would it cost to insure an automobile against fire for $300?

2. In the same city, the rate is 40¢ a hundred to insure a new car against fire. How much will it cost to insure a new automobile for $1500 against fire?

3. The rate of insurance against theft in that city for the car mentioned in example 2 is $1.30 per hundred. How much will it cost to insure the machine for $1500 against theft

4. The liability cost in that city to cover injuries or death to one person to the limit of $10,000 and injuries or death to more than one person to the limit of $20,000 is $128.40. How much is that per $1000 in case of death or injury to one person for the full limit?

Fire Insurance

On the day that Jim investigates insurance on the house the following points were brought up and the questions that were raised led to the following problems.

The Arithmetic of Fire Insurance

There are three elements involved in problems on insurance, viz. the amount insured, the rate, and the premium. The amount is the base in percentage, the rate of insurance is the rate, and the premium is the percentage.

The rate is fixed by the Company according to the probability of loss as determined by the collective experience of all insurance companies. When the amount of insurance and the rate are known, the finding of the premium is the simplest case in percentage.

Example 1. Household goods valued at $5000 are insured at the rate of 50¢ per $100. What is the premium for a three year period?

Solution: The premium for one year is $25.00
Premium for 3 years=2½x$25.= 62.50

Example 2. An office building valued at $175,000 is insured under the average clause for $140,000. A fire damages the building to the extent of $12,000. What is the amount of insurance paid?

Solution: The ratio of the total insurance to the value of the building is $\frac{140000}{175000} = \frac{4}{5}$ Hence the insurance paid is $\frac{4}{5} \times 1200 = \$9600$.

Fire insurance is usually paid in advance for a term of years. The

amount of the premium for a 3 year policy is usually 2½ times the premium for a 1 year policy; the premium on a 5 year policy is usually 4 times the premium for a 1 year policy.

Exercises

1. What is the premium for (a) a 3 year policy? (b) a 5 year policy, if the rate on a 1 year policy is 34 cents? 62cents? $1.14? $1.46?

2. What is the premium for (a) a 3 year policy? (b) a 5 year policy, if the premium on a 1 year policy is $14.42? $8.68? $39.89? $126.14?

3. Jim then revisited the automobile dealer and was given the following problems:

    a. An automobile insured against fire for $1,800 at the rate of 45¢ per $100 per year. How much will the insurance cost him?

    b. He also learned that fire insurance is more expensive on a garage than on a house? Can you tell him why?

Prepare the solutions to the following questions raised in the preceding discussion:

*4. What are the rates for fire and automobile insurance for your city?

Problems

1. A man owned 2/5 of a shop and insured 3/4 of his interest at 1¼%, paying $112.50 premium. What was the value of his interest in the shop? If the shop was damaged to the extent of $4,000, what did he receive from the insurance company?

2. A mill worth $30,00 was insured for an annual premium of 1-2/3% on 90% of its value. In the second year it was injured by fire to the amount of $1175. How much did the mill owner save by insuring?

3. The premium on a building, 5/8 of which is insured at 1-1/3% is $25.50. What is the value of the building?

4. Find the cost of insuring a grain elevator for $12,000 for 3 months if the annual rate is 5/8%?

5. A house valued at $4500 was insured for 2/3 of its value for 1 year at 60¢ per $100. At the end of 7 months the house was sold and the policy was cancelled by the insured. How much should be returned?

6. Find the cost of insuring stored wheat valued at $310,000 for 3 months at 80¢ per $100 per annum.

7. Find the cost of insuring a factory for $10,000 for 5 years if the annual premium is 3/8%.

National Service Life Insurance

The final lap of Jim's journey was made at the Veterans Administration headquarters. He waited to go here last because he already had in his possession several booklets given him while he was in service. On the next page there are a few of these pages that will be studied for comparison with other insurances studied.

The policies are written in multiples of $1,000, the minimum being $1,000 and the maximum $10,000. After the war, it was optional whether to continue the insurance or not. Any insured veteran might carry an insurance from $1,000 up to the amount he carried during the war, and in multiples of $500. These converted policies are in the form of ordinary life, twenty payment life, thirty-year endowment, endowment at age 62, and five-year level premium term policies.

On the following page is a table from the recent bulletin given to Jim that was published by the United States Government. Regardless of what kind of policy this may be compared with, it is by far the most beneficial, so that for a veteran, there is none better.

Can you answer the following question for Jim?

1. Name several reasons why this type of insurance is cheaper to the

5. Choose values of £500 are insured for £1½ if the period for 1 year at £24 per £100. In the case of 7 months the insurance would be £1 plus the interest on the insurance. How much should be paid?

6. Find the cost of insuring stock valued at £10,000 for 1 months at £24 per £100 per annum.

7. Find the cost of insuring a factory for £20,000 for 9 months if the annual premium is £½%.

Technical Drawing and Drawings

The first leg of Jim's journey was made at the Telecom Manufacturing Headquarters. He waited to be more just before he finally met in the research a several bookings given him skill to use it better. On the next page there are a few of those pages that will be studied for accuracy- sor with other insurances studied.

The building are written in multiples of £1,000, the minimum being £1,000 and the maximum £10,000. After the war, it was optional whether to continue the insurance or not. Any insured veteran might apply an insurance item £1,000 up to the amount he carried during the war, and in multiples of £500. Those concerned policies are in the form of ordinary life, twenty payment life, thirty-year endowment, endowment at age 65, and five-year level premium term policies.

On the following page is a table from the recent building sites in the United States Government. Technicians at most time of doing this may be concerned with, it is so far the most beautiful, so that for a moment, there is none below.

Can you answer the following question for Jim?

1. Jim wanted renewing his this type of insurance to change at the

veteran than any other type?

*2. Take a specific age and a specific kind of life insurance and compare from any of the references listed the information given for the same or similar policy listed in this table.

Aside from insurance the Browns considered other forms of investment. Let us solve the following questions raised in this connection.

Investments

There are many types of investments aside from home and life insurance. The next step then is toward the investment in stocks and bonds. The different kinds of shares and stocks will not be discussed fully here but the pupil is recommended to Supplementary text, especially Economic Mathematics by Sutton and Lerves.

United States Government Bonds are classed as the safest of all investments because of the faith of the American people in the integrity and stability of the Government. Government bonds are non-taxable.

It is this type of bond that will be studied fully here.

I. Bring in current advertisements on Savings Bonds issued by the U. S. Government.

II. Why is this considered the safest investment in bonds today?

III. Explain the process of converting saving stamps into bonds?

IV. Describe in detail the face of a Saving Bond.

Stocks and Bonds

1. A United States Saving Bond costs $18.75. Its maturity value is $25.00. Is this a wise investment? If the bond reaches its

PLAN	Premium per $1,000 of insurance at age 25			
	Monthly	*Quarterly*	*Semi-annual*	*Annual*
20-year endowment	$3.48	$10.41	$20.75	$41.20
Endowment at age 60	1.89	5.66	11.27	22.38
Endowment at age 65	1.67	5.00	9.96	19.77

Guaranteed Values at Age 25—$1,000

20-YEAR ENDOWMENT

End of policy year	Cash value	Paid-up insurance	Extension		Pure endowment	End of policy year	Cash value	Paid-up insurance	Extension		Pure endowment
			Years	Days					Years	Days	
1	$34.45	$57.51	4	219	------	11	$457.41	$590.47	9	---	$547.46
2	70.16	114.30	9	345	------	12	508.90	639.28	8	---	605.94
3	107.19	170.35	16	10	------	13	562.37	687.28	7	---	662.21
4	145.58	225.65	16	---	$68.76	14	617.88	734.43	6	---	716.35
5	185.39	280.18	15	---	145.19	15	675.54	780.75	5	---	768.40
6	226.67	333.91	14	---	218.78	16	735.46	826.23	4	---	818.47
7	269.49	386.85	13	---	289.63	17	797.73	870.88	3	---	866.59
8	313.91	438.99	12	---	357.82	18	862.49	914.73	2	---	912.85
9	359.98	490.30	11	---	423.46	19	929.87	957.77	1	---	957.31
10	407.79	540.79	10	---	486.65	20	1,000.00	1,000.00	---	---	------

ENDOWMENT AT AGE 60

1	$14.99	$33.93	1	339	------	13	$240.41	$422.15	22	---	$133.25
2	30.49	67.68	4	---	------	14	263.63	452.47	21	---	191.90
3	46.52	101.22	6	83	------	15	287.64	482.41	20	---	248.30
4	63.10	134.54	8	220	------	16	312.46	511.95	19	---	302.49
5	80.25	167.63	11	37	------	17	338.15	541.14	18	---	354.68
6	97.99	200.48	13	247	------	18	364.71	569.91	17	---	404.77
7	116.34	233.07	16	88	------	19	392.19	598.30	16	---	452.90
8	135.32	265.37	18	240	------	20	420.61	626.26	15	---	499.17
9	154.95	297.38	20	303	------	25	578.27	759.90	10	---	704.06
10	175.24	329.06	22	262	------	30	766.78	883.57	5	---	869.43
11	196.24	360.43	24	---	$8.91	35	1,000.00	1,000.00	---	---	------
12	217.95	391.46	23	---	72.31						

ENDOWMENT AT AGE 65

1	$12.30	$29.96	1	210	------	13	$195.99	$376.78	22	229	------
2	25.01	59.81	3	91	------	14	214.75	404.14	23	211	------
3	38.15	89.56	5	10	------	15	234.12	431.22	24	141	------
4	51.72	119.15	6	330	------	16	254.11	457.98	24	---	$41.74
5	65.75	148.61	8	322	------	17	274.74	484.41	23	---	104.40
6	80.24	177.88	10	334	------	18	296.03	510.50	22	---	164.66
7	95.21	206.96	12	357	------	19	318.01	536.28	21	---	222.56
8	110.68	235.86	15	5	------	20	340.68	561.70	20	---	278.22
9	126.66	264.54	16	341	------	25	465.11	683.20	15	---	524.62
10	143.16	292.98	18	214	------	30	610.09	795.11	10	---	723.51
11	160.21	321.18	20	69	------	35	782.35	899.14	5	---	880.58
12	177.81	349.11	21	185	------	40	1,000.00	1,000.00	---	---	------

The above values are based on an insurance of $1,000 without indebtedness and with no dividends standing to the credit of the policy.

The 5-year level premium term policy has no cash, paid-up insurance or extended insurance values.

sion

Additional premiums for the Total Disability Income Provision, as authorized by the Insurance Act of 1946, are payable in the same manner and at the same time as premiums on the life or endowment insurance policy to which it is attached, either to age 60 or to the end of the premium-paying period of the insurance policy whichever is the shorter premium-paying period.

PREMIUM RATES PER $1,000 OF INSURANCE

Rates in tables 1 to 5, inclusive, are based on the date the disability provision becomes effective.

TABLE 1

5-year term, ordinary life, endowments at ages 60 and 65

[These rates apply whether the disability provision is added to new insurance or to policies in force]

TABLE 2

20-payment life

[These rates apply only when the disability provision is added to new insurance. They *do not* apply when the provision is added to a policy in force—in such cases see special rates in table 6]

Monthly	Quarterly	Semiannual	Annual	AGE at nearest birthday	Monthly	Quarterly	Semiannual	Annual
$0.16	$0.48	$0.95	$1.89	15	$0.24	$0.72	$1.43	$2.84
.16	.48	.95	1.89	16	.24	.72	1.43	2.84
.16	.48	.95	1.89	17	.24	.72	1.43	2.84
.16	.48	.95	1.89	18	.25	.75	1.49	2.96
.17	.51	1.01	2.01	19	.25	75	1 49	2 96
.17	51	1 01	2 01	20	26	.78	1.55	3.08
.18	.54	1.07	2.13	21	.26	.78	1.55	3.08
.19	.57	1.13	2.25	22	.27	.81	1.61	3.20
.19	.57	1.13	2.25	23	.27	.81	1.61	3.20
.20	.60	1.19	2.37	24	.28	.84	1.67	3.31
.21	.63	1.25	2.49	25	.28	.84	1.67	3.31
.21	.63	1.25	2.49	26	.29	.87	1.73	3.43
.22	.66	1.31	2.60	27	.30	.90	1.79	3.55
.23	.69	1.37	2.72	28	.30	.90	1.79	3.55
.24	.72	1.43	2.84	29	.31	.93	1.85	3.67
.25	.75	1.49	2.96	30	.31	.93	1.85	3.67
.25	.75	1.49	2.96	31	.32	.96	1.91	3.79
.26	.78	1.55	3.08	32	.32	.96	1.91	3.79
.28	.84	1.67	3.31	33	.33	.99	1.97	3.91
.29	.87	1.73	3.43	34	.33	.99	1.97	3.91
.30	.90	1.79	3.55	35	.34	1.02	2.03	4.03
.31	.93	1.85	3.67	36	.35	1.05	2.09	4.14
.32	.96	1.91	3.79	37	.35	1.05	2.09	4.14
.34	1.02	2.03	4.03	38	.36	1.08	2.15	4.26
.35	1.05	2.09	4.14	39	.36	1.08	2.15	4.26

For rates at age 40 and above, see table 5

ENDOWMENT AT AGE 60

Age at nearest birthday	Premium per $1,000			
	Monthly	Quarterly	Semi-annual	Annual
15	$1.40	$4.19	$8.35	$16.57
16	1.44	4.31	8.59	17.05
17	1.48	4.43	8.83	17.52
18	1.52	4.55	9.06	18.00
19	1.57	4.70	9.36	18.59
20	1.61	4.82	9.60	19.06
21	1.66	4.97	9.90	19.65
22	1.71	5.12	10.20	20.24
23	1.77	5.30	10.55	20.95
24	1.83	5.48	10.91	21.67
25	1.89	5.66	11.27	22.38
26	1.96	5.87	11.69	23.20
27	2.03	6.08	12.11	24.03
28	2.10	6.28	12.52	24.86
29	2.18	6.52	13.00	25.81
30	2.27	6.79	13.54	26.87
31	2.36	7.06	14.07	27.94
32	2.47	7.39	14.73	29.24
33	2.57	7.69	15.33	30.43
34	2.69	8.05	16.04	31.85
35	2.82	8.44	16.82	33.39
36	2.96	8.86	17.65	35.04
37	3.11	9.31	18.55	36.82
38	3.27	9.79	19.50	38.71
39	3.46	10.35	20.63	40.96
40	3.66	10.95	21.83	43.33
41	3.88	11.61	23.14	45.94
42	4.13	12.36	24.63	48.89
43	4.40	13.17	26.24	52.09
44	4.71	14.10	28.09	55.76
45	5.07	15.17	30.23	60.02
46	5.47	16.37	32.62	64.76
47	5.94	17.78	35.42	70.32
48	6.48	19.39	38.64	76.72
49	7.13	21.34	42.52	84.41
50	7.90	23.64	47.11	93.53
51	8.84	26.45	52.71	104.66
52	10.01	29.96	59.69	118.51
53	11.51	34.45	68.64	136.27
54	13.52	40.46	80.62	160.06
55	16.31	48.81	97.26	193.09
56	20.49	61.32	122.19	242.58
57	27.45	82.15	163.69	324.98
58	41.34	123.72	246.52	489.42
59	82.94	248.21	494.59	981.92

Premium Rates—*Continued*

ENDOWMENT AT AGE 65

Age at nearest birthday	Premium per $1,000			
	Monthly	Quarterly	Semi-annual	Annual
15	$1.28	$3.83	$7.63	$15.15
16	1.31	3.92	7.81	15.51
17	1.35	4.04	8.05	15.98
18	1.38	4.13	8.23	16.34
19	1.41	4.22	8.41	16.69
20	1.45	4.34	8.65	17.17
21	1.49	4.46	8.89	17.64
22	1.53	4.58	9.12	18.11
23	1.58	4.73	9.42	18.71
24	1.62	4.85	9.66	19.18
25	1.67	5.00	9.96	19.77
26	1.72	5.15	10.26	20.36
27	1.78	5.33	10.61	21.07
28	1.84	5.51	10.97	21.78
29	1.90	5.69	11.33	22.49
30	1.96	5.87	11.69	23.20
31	2.03	6.08	12.11	24.03
32	2.11	6.31	12.58	24.98
33	2.19	6.55	13.06	25.93
34	2.27	6.79	13.54	26.87
35	2.37	7.09	14.13	28.06
36	2.46	7.36	14.67	29.12
37	2.57	7.69	15.33	30.43
38	2.69	8.05	16.04	31.85
39	2.81	8.41	16.76	33.27
40	2.94	8.80	17.53	34.81
41	3.09	9.25	18.43	36.58
42	3.25	9.73	19.38	38.48
43	3.42	10.23	20.39	40.49
44	3.61	10.80	21.53	42.74
45	3.82	11.43	22.78	45.22
46	4.06	12.15	24.21	48.07
47	4.32	12.93	25.76	51.14
48	4.61	13.80	27.49	54.58
49	4.93	14.75	29.40	58.37
50	5.30	15.86	31.61	62.75
51	5.72	17.12	34.11	67.72
52	6.20	18.55	36.97	73.40
53	6.76	20.23	40.31	80.03
54	7.42	22.21	44.25	87.85
55	8.21	24.57	48.96	97.20
56	9.17	27.44	54.68	108.56
57	10.36	31.00	61.78	122.65
58	11.88	35.55	70.84	140.65
59	13.90	41.60	82.89	164.56
60	16.72	50.04	99.71	197.95
61	20.92	62.61	124.75	247.67
62	27.90	83.49	166.37	330.31
63	41.80	125.09	249.26	494.87
64	83.41	249.62	497.39	987.49

SECTION XIII.—Guaranteed Values on National Service Life Insurance

PLAN	Premium per $1,000 of insurance at age 25			
	Monthly	*Quarterly*	*Semi-annual*	*Annual*
Ordinary Life	$1.37	$4.10	$8.17	$16.22
30-Payment Life	1.67	5.00	9.96	19.77
20-Payment Life	2.12	6.34	12.64	25.10

Guaranteed Values at Age 25—$1,000

ORDINARY LIFE

End of policy year	Cash value	Paid-up insurance	Extension		End of policy year	Cash value	Paid-up insurance	Extension	
			Years	Days				Years	Days
1	$8.60	$23.78	1	34	13	$134.77	$304.26	15	355
2	17.47	47.55	2	87	14	147.39	326.76	16	297
3	26.61	71.28	3	158	15	160.36	349.05	17	190
4	36.04	94.99	4	249	16	173.67	371.09	18	41
5	45.76	118.66	5	354	17	187.34	392.91	18	215
6	55.77	142.24	7	111	18	201.37	414.49	18	352
7	66.09	165.75	8	240	19	215.77	435.81	19	91
8	76.72	189.16	10	6	20	230.50	456.82	19	165
9	87.67	212.47	11	133	25	309.14	556.79	19	197
10	98.94	235.64	12	244	30	394.11	646.17	18	220
11	110.55	258.68	13	325	35	482.33	723.44	17	52
12	122.49	281.56	14	364	40	570.12	788.29	15	145

30-PAYMENT LIFE

1	$12.30	$34.00	1	210	13	$195.94	$442.35	22	227
2	25.01	68.07	3	91	14	214.70	475.98	23	210
3	38.14	102.16	5	9	15	234.06	509.47	24	139
4	51.71	136.30	6	329	16	254.04	542.82	25	24
5	65.73	170.44	8	321	17	274.67	576.07	25	234
6	80.22	204.60	10	333	18	295.96	609.18	26	50
7	95.19	238.73	12	356	19	317.93	642.15	26	206
8	110.66	272.84	15	4	20	340.59	675.00	26	343
9	126.63	306.89	16	340	25	464.99	837.49	28	286
10	143.13	340.88	18	242	30	609.92	1,000.00	------	-----
11	160.17	374.79	20	68	35	666.72	--------	------	-----
12	177.77	408.63	21	183	40	723.24	--------	------	-----

20-PAYMENT LIFE

1	$17.81	$49.24	2	110	13	$287.07	$648.09	31	51
2	36.24	98.63	4	294	14	314.97	698.27	32	126
3	55.31	148.16	7	193	15	343.86	748.47	33	188
4	75.06	197.84	10	166	16	373.77	798.65	34	263
5	95.49	247.61	13	195	17	404.76	848.91	36	27
6	116.64	297.48	16	241	18	436.85	899.18	37	269
7	138.54	347.45	19	236	19	470.12	949.55	40	40
8	161.21	397.48	22	121	20	504.58	1,000.00	------	-----
9	184.66	447.52	24	237	25	555.22	--------	------	-----
10	208.95	497.64	26	232	30	609.92	--------	------	-----
11	234.09	547.76	28	124	35	666.72	--------	------	-----
12	260.12	597.92	29	300	40	723.24	--------	------	-----

The above values are based on an insurance of $1,000 without indebtedness and with no dividends standing to the credit of the policy.

The 5-year level premium term policy has no cash, paid-up insurance, or extended insurance values.

TOTAL DISABILITY PREMIUM RATES—Contd.

TABLE 7.—*20-year endowment—$1,000*

MONTHLY PREMIUMS

Age at issue of existing policy	Number of months insurance policy has been in force									
	1 but less than 13	13 but less than 25	25 but less than 37	37 but less than 49	49 but less than 61	61 but less than 73	73 but less than 85	85 but less than 97	97 but less than 109	109 but less than 121
15	$0.06	$0.06	$0.07	$0.07	$0.07	$0.07	$0.07	$0.08	$0.08	$0.08
16	.07	.07	.07	.07	.07	.08	.08	.08	.08	.08
17	.07	.07	.07	.07	.08	.08	.08	.08	.08	.09
18	.07	.07	.08	.08	.08	.08	.08	.09	.09	.09
19	.08	.08	.08	.08	.08	.09	.09	.09	.09	.10
20	.08	.08	.08	.09	.09	.09	.09	.10	.10	.10
21	.08	.09	.09	.09	.09	.10	.10	.10	.10	.11
22	.09	.09	.09	.10	.10	.10	.10	.11	.11	.11
23	.09	.10	.10	.10	.11	.11	.11	.11	.12	.12
24	.10	.10	.11	.11	.11	.12	.12	.12	.13	.13
25	.11	.11	.11	.12	.12	.12	.13	.13	.13	.14
26	.11	.12	.12	.12	.13	.13	.14	.14	.15	.15
27	.12	.13	.13	.13	.14	.14	.15	.15	.16	.16
28	.13	.14	.14	.14	.15	.15	.16	.17	.17	.18
29	.14	.15	.15	.16	.16	.17	.17	.18	.19	.20
30	.15	.16	.16	.17	.18	.18	.19	.20	.21	.22
31	.17	.17	.18	.19	.19	.20	.21	.22	.23	.24
32	.18	.19	.20	.20	.21	.22	.23	.24	.25	.27
33	.20	.21	.22	.22	.23	.24	.26	.27	.28	.29
34	.22	.23	.24	.25	.26	.27	.28	.30	.31	.33
35	.24	.25	.26	.27	.29	.30	.31	.33	.35	.36
36	.26	.27	.29	.30	.31	.33	.35	.36	.38	.40
37	.29	.30	.32	.33	.35	.37	.38	.40	.42	.45
38	.32	.33	.35	.37	.38	.40	.42	.45	.47	.49
39	.35	.37	.39	.40	.42	.45	.47	.49	.52	.55

For rates at ages 40 to 59, inclusive, see table 5

TABLE 8.—*30-payment life—$1,000*

MONTHLY PREMIUMS

Age at issue of existing policy	Number of months insurance policy has been in force									
	1 but less than 13	13 but less than 25	25 but less than 37	37 but less than 49	49 but less than 61	61 but less than 73	73 but less than 85	85 but less than 97	97 but less than 109	109 but less than 121
15	$0.19	$0.19	$0.20	$0.21	$0.22	$0.23	$0.24	$0.26	$0.27	$0.28
16	.19	.20	.21	.22	.23	.24	.25	.26	.27	.29
17	.19	.20	.21	.22	.23	.24	.25	.27	.28	.30
18	.20	.21	.22	.23	.24	.25	.26	.27	.29	.30
19	.20	.21	.22	.23	.24	.25	.26	.28	.29	.31
20	.21	.22	.23	.24	.25	.26	.27	.28	.30	.31
21	.21	.22	.23	.24	.25	.26	.27	.29	.30	.32
22	.22	.23	.23	.24	.26	.27	.28	.29	.31	.32
23	.22	.23	.24	.25	.26	.27	.28	.30	.31	.33
24	.23	.23	.24	.25	.27	.28	.29	.30	.32	.33
25	.23	.24	.25	.26	.27	.28	.30	.31	.32	.34
26	.23	.24	.25	.26	.28	.29	.30	.32	.33	.35
27	.24	.25	.26	.27	.28	.29	.31	.32	.34	.35
28	.24	.25	.26	.28	.29	.30	.31	.33	.34	.36
29	.25	.26	.27	.28	.29	.30	.32	.33	.35	.36

For rates at ages 30 to 59, inclusive, see table 4 or 5

maturity after 10 years, what is rate of interest for that period?

2. Marjorie Gray bought a United States Savings Bond. She is to receive $100 for the bond at the end of 10 years. If she sells it before it matures (before the end of the 10-year period), she will receive the purchase price with compound interest at a rate of nearly 3%. Do you consider this a wise investment?

3. Why doesn't the Government pay interest on war bonds every year instead of letting the interest accumulate in the increased value of a bond?

4. Is there any limit to the amount that may be invested in war bonds in any one year by one person? by a family?

5. What interest does the savings bank in your community pay?

6. How would you compute the interest on $1000 for 2 years compounded semi-annually at 2%? Work it out.

Cumulative Review

1. What is meant by merchant's "Mark-up"? In deciding on rate percent of mark-up, what is used as the base? In finding the net percent margin what is used as the base? What must be the mark-up rate to make the margin 30%.

2. The value of a car at the beginning of the year is $720. The depreciation during the year includes: 982 gallons of gasoline at an average price of 19½¢ per gallon; 31 quarts of oil @ 29¢ per quart; new tires $74.60; anti-freeze $4.75; license $10; tax $7.85; repairs $49.70; insurance $6130; garage $60. What is the average cost per mile of driving this car if it was driven 13,910 miles?

3. Geneva Cash left high school at the close of her first year and obtained a position at $10 a week; the second year she received $11 a week; and for the next eight years she worked at $13. a week. Alva Sharpe, who entered high school the same year, finished the Commercial Course and obtained a position as a stenographer. The first year she received $15 a week; the second year $18 a week; and the next three years $20 a week; and the next two years $25 a week.

   (a) Make a chart with horizontal columns headed years, Geneva and Alva; in the column headed years, write 1-10 vertically.

   (b) Fill in all spaces in the chart.

   (c) Enter in correct place the total amount each girl earned during the ten year period.

   (d) How much more did Alva earn than Geneva during the ten years?

   (e) How much did Geneva earn while Alva was completing high school?

Miscellaneous Types of Insurance

Due to a limited amount of both time and space all types of insurance will not be considered here but may be introduced as special reports for students desiring to make up or do supplementary work. These include:

a. Professional Liability Insurance
b. Savings bank insurance
c. Compensation insurance
d. Industrial policies
e. Rain and hail insurance
f. Tornado insurance

4. Answer only TWO parts of the three in this question.

(a) Write a short note on Ethical Dilemma in Business.

(b) Explain the concept of Corporate Governance.

(c) Write a short note on Corporate Social Responsibility.

g. Hurricane insurance
h. Group life insurance

These many kinds of insurance, which the children should know about in order to understand the kinds of policies they will need to use in computation, form an important part of the pupil's informational background.

A grave error that an insured person should guard against is leaving to his beneficiary, if he or she has had no training in the handling and investing of money, the entire insurance in one sum. Such a person frequently becomes the victim of swindlers and the purpose of the insurance is defeated. "Authorities have established the fact that one-third of the vast sums paid in death claims is dissipated through the beneficiaries' inexperience, improvidence, or bad judgement."[1/]

Mastery Test on Insurance

Directions: Write the numbers 1-20 in a column. Place a plus after each number whose corresponding statement is true. Place a zero after each number whose corresponding statement is false or partially false.

1. The rate of a five year insurance policy is five times the annual rate _____.
2. Endowment insurance provides protection only _____.
3. Life insurance is a speculation _____.
4. Statements made in the application for life insurance must be truthful _____.
5. Fire insurance claims are always settled for the face of the policy _____.

1/ Sylvester Dunham, The Business of Insurance, P. 300, Ronald Press Co., New York, 1912.

5. Insurance Insurance
6. Income Life Insurance

There many kinds of insurance, which the children should learn about in order to understand the kinds of policies they will need to use in operations. Here an illustration used by Mrs Gould's Industrial Insurance.

A set of cards that an insured person should have with him in case of accident, if he is also not able to explain to the doctor and hospital of course, his policy insurance information. Such a system frequently becomes the victim of neglect and the necessary can [illegible] is omitted. "Hospitalia form originated the term [illegible] [illegible] [illegible]

6. It is never wise to carry more fire insurance than the value of the thing insured _____.

7. Fire insurance rates vary for different types of buildings in different sections of the city _____.

8. The cost of a $6,000 fire insurance policy at 45¢ per $100 would be $30.00 _____.

9. Rates for collision insurance are high _____.

10. Term insurance is a good type of permanent insurance _____.

11. The net rates of all insurance companies are the same _____.

12. Fire insurance on a building covers the building only _____.

13. Part of one's expenditure for life insurance represents an investment _____.

14. All veterans do well to hold on to their "G. I." policies _____.

15. Ordinary life insurance is 100% protection _____.

16. A life insurance policy is the same as a will _____.

17. The cost of fire insurance is an asset _____.

18. Collision insurance provides protection against damage to one's car _____.

19. A life insurance company loans money only to policy holders owning real estate _____.

20. Life insurance is a good protection for one's family _____.

At the end of Jim's "insurance tours", he was quite pleased with his discoveries but as he told Mrs. Brown about the conversations, he was a bit puzzled by the terms most of the agents used in relation to related values. Either the related value of age to the premium or of one insurance to the other.

He never before felt the need of retaining these facts that he remembers only vaguely. He asks Mrs. Brown to include in her course for her

class, and to refresh his memory more about the equation and formula.

The following section is devoted to a study of the principles involved in these phases of algebra. The text book referred to in your instruction will be your third reference - General Mathematics by Edgerton and Carpenter.

Formulas and Equations

Study and Practice

Related Numbers

When one number is dependent upon a second number, the two are related, and the first is a function of the second. For instance, the circumference of a circle changes when the diameter increases or decreases. That is, the circumference and the diameter are definitely related. A number may be dependent upon more than one other number. For example, the area of a rectangle is dependent not only upon the length but also upon the width; or we may say the area of a rectangle is dependent upon the product of the length and the width. In this case, the area, the length, and the width are related numbers.

Related numbers may be illustrated by tables showing how one number changes when there is a change in another number on which the first depends. From a study of these changes we can write a formula from the table.

For example, a certain motorboat travels at an average rate of 35 miles an hour. The following table shows the relation between the hours and the distance travelled.

Miles	35	70	105	140	175	210	245	280	315	350
Hours	1	2	3	4	5	6	7	8	9	10

Exercise for Practice

Letter to Friends

Central College

[illegible] January 1[illegible]

Dear Friends,

I am very glad to hear that you are coming, and the first week in February will be very convenient. Do come to dinner on the first day, and then we can arrange what we will do afterwards. I think it will be best for you to stay with us, as then you will be near the college, and can easily get to lectures. Let me know by return of post what train you will come by, and I will meet you at the station.

Yours sincerely,

Alice Jones.

Solution: The numbers in each column of the table are related to as 1 to 35. By studying the table we notice that in each case the distance can be found by multiplying the number of hours by 35. If we let t equal the number of hours, the formula for the distance may be written:

$$d = 35t$$

Study: page 60 and the illustrative example on page 61 of your textbook.

Work: Exercises 1, 2, 3, and 7 on page 61 and Exercise 11 on page 63 of your textbook.

Writing Formulas from the Related Numbers of a Table

Study: the material given under Writing Formulas from the Related Numbers of a Table, pages 63 and 64 of your textbook.

Let us examine Exercise 1 on page 64 of your textbook. "As the number of pounds increases in order, the cost increases 2 cents a pound." If we start with 7¢ and add 2¢ for the first pound, we will have 9¢, 11¢ and 13¢, as given in the table. Therefore, if we let c represent the cost and w the weight in pounds, the formula may be written:

$$c = 2w + 7$$

Work: Exercises 3, 4, 5, 6, and 7 on page 65 of your textbook.

Formulas as General Solutions

This topic is explained quite simply in your textbook.

The Equation

The equation is the most important of all algebraic expressions; in fact, algebra is sometimes defined as the science of the equation. An equation is a statement in symbols of the fact that two algebraic expressions are equal.

We shall take up briefly the main types of simple equations.

Solution: The number in each column of the table is related to
on 1 by 3's. By starting the table on 1 instead of 0, we can use the table
can be found by multiplying the number of boxes by 3. So the 4 small
box number of boxes, the formula for the difference can be written:

$d = 3n$

Check: Does 30 fit the illustrative formula: on page 61 of your textbook.

Does: Determine d, n, and T so that $T = d + n$ is true for
of your textbook.

Writing formulas from the verbal sentence of a topic

Study: In mathematics, many topics formulas from verbal sentences
to a result, spread it out by very carefully.

Let us consider Question 1 on page 64 of your textbook. "In the country
a certain interest in debt, the debt increasing 5% during a year." If an
owed $1/4 and was not $4 for the first year, he still owe 95% of $1, 11% and 11%,
he given in the table. Solution, If We let n represent the part and T the
whole in cents, the formula was to write:

$n + 5n = T$

Study: Determine n, 5, 6, and T on page 64 of your textbook.

Computer or Computational Activities

This block is exclusively style in your textbook.

The Question

The question is the most important of all algebra word-problems; it
that, despite its confusing setting as the science of all quality. In
addition to a discussion in symbol or the text that was explain, consumers
may exist.

We shall name of Julie's the most types of algebra word-problem.

Study: pages 77 and 78 of your textbook.

Type I. $ax = b$

In this type of equation, a and b will have different values, each different value for a or b making a different equation. Examples of this type are: $3x = 9$; $2x = 2$; $6x = 24$; etc.

Study: the illustrative example given on page 78 of your textbook. Remember that the solution of an equation is never finished until the answers are checked or proved.

Work: Exercises 2, 3, 4, and 6 on page 78 of your textbook.

Problems Applying Type I

Many students find that the hardest part in the solution of problems is setting up the equations. We shall set up the equation for one of the problems similar to those on page 79 of your textbook. You should have no difficulty in solving the equation, once it has been set up.

Problem: Five times John's score equals 400. Find his score.
Let x = his score then 5 times the score will be 400:
Hence $5x = 400$ (the required equation)

You should complete the solution by solving for x and checking your answer.

Solve: Exercises 1 to 6 at the top of page 80 of your textbook.

Self-Check Test

Work: the practice test in solving equations and the Selection Test given on page 87 of your textbook.

Vocabulary Test

Test yourself to make sure that you know the meaning of all the terms given in the list on page 89 of your textbook.

Type 1: Items 71 and 72 of your textbook.

Type 2: ax = b

In this type of equation, a and b will have different values, each different value for x or b making a different problem. Examples of this type are: $4x = 7$; $5x = 2$; $-3x = 9$; etc.

Type: The multiplicative property rules on Item 73 of your textbook. Remember that the solution of an equation is never finished until the answer has checked by method.

Note: Examine $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ on page 73 of your textbook.

Problems dealing Type 1

Many students find that the decimal part in the solution of problems in solving by the equation. To deal out to the question for one of the problems similar to those on page 78 of your textbook. You should have no difficulty in solving the equation, once it has been set up.

Problem: Five times John's young brother 600. Find his age.
Let x this equation when 5 times the other will be 600:
Solve for x and find the required equation)

The lesson develops the ability to solve the x-two character for

Note: Remember to do a at the of step 3 of your textbook.

Self-Test Set

Note: The question next is solving equations and the directions are given on page 77 of your textbook.

Evaluation Item

Find Present is some way that can have the method of all the items given in the list on page of your textbook.

F. Commentary on The Unit Insurance

As children go into the business world, they should understand that in the development and conduct of any business, insurance is a major item. The average child in junior high school knows something of insurance. Life insurance, Term insurance, Annuities, Veterans, Automobile, Fire, Health and Accident insurance so meet the needs of everyday life that they are discussed in the ordinary home. These children are very apt to have personal experience in the home in paying premiums and receiving insurance money. With the right teaching technique, the problem of insurance should appeal strongly to his interest. Since the insurance companies do practically all the computation, the teaching time will be spent largely on the informational and appreciative side of the problem.

It is the aim here to have a good understanding of what the term connotes. At the end of this unit, the pupil will make up his own description of the term.

Motivation and provision for individual differences will be the general techniques of this unit. Socialization of this work will be an outstanding necessity in this development of the unit. Questioning in the appreciation technique must be skillfully done. The wrong kind of questioning may defeat the purpose of appreciation. The "why", the "yes" and the "no" type will be avoided. The aim is to have every child happy, interested and progressing to the extent of his ability. Supervised study is essential in this insurance unit. The Teacher's obligation is to supervise the placing of the right type of material in the pupil's hands.

In this unit on insurance, the social phase of the work will be used for

motivation and stimulation. However, the mathematics needed for this and similar topics will not be neglected. It is hoped to progress from the simple arithmetic computations to the wider applications of principles in algebra so naturally that horizons will be enlarged without great efforts on the part of the pupil.

To go from familiar to new ideas in the simple graph will enable the use of graphs as an introduction to algebra. The work on graphs will begin with the simpler forms but it is intended to work mostly with graphs of equations and formulas. Here again the problem of individual difference will be taken care of. Beginning with the bar graphs for the slow learner, the more apt will go on to such graphs of equations and formulas involving the plotting of coordinates. He will graph the meaning of independent and dependent variable, the advantages and disadvantages of investing savings and drawing graphs of the annual yield of an investment. There will be committees reporting upon graphical representation readings from current periodicals brought in for reading and interpretation, emphasizing the dangers of misrepresentation in the pictograph.

The main theme of the unit will be the part insurance plays in our everyday life so that the topics will come up incidentally under insurance.

CHAPTER V

FINAL REMARKS ON BOTH UNITS

Conclusions.-- Although no final conclusion can be reached concerning these units, it may be stated that this is just a beginning. Future units will be constructed around these social topics instead of around algebraic ones until finally a whole course of study will be presented in this manner.

Limited Time Available.-- A more comprehensive study is now impossible because of such limited time, but great effort has been made to improve the course from the standpoint of the selection and organization of the materials composing these units. Because of such little time much empiricism is employed in applying the criteria and principles involved. It was necessary to rely rather heavily on adaptations of the results of investigations made and reported by other mathematics teachers.[1/]

From current literature it appears that pupils who are properly stimulated respond to principles of algebra better than those who are taught by the rigid methods once employed in the Latin Grammar Schools.

Revisions Made.-- The traditional course in algebra started with such topics as the formula, the graph, the equation, and positive and negative numbers. Centering around these, the everyday practical situations were brought in under an avalanche of rules and processes. In this course an everyday topic such as insurance is the core of the unit while algebraic

1/ Roy O. Billett, Fundamentals of Secondary School Teaching, Houghton Mifflin Company, Boston, 1940, P. 172.

CHAPTER V

OTHER ELEMENTS IN BODY DESIGN

Construction.— Economy of final construction can be secured sometimes only, it may be stated that this is just a beginning. Factory methods will be considered before that time. Local fabrics insured a record electric cars will effect a short period of study will be necessary in this manner.

Light type existing.— A more thoroughgoing study in the improved passing or even limited time, but great effort has been made to increase the car weight the reduction of the capacity and maintenance of the mechanical connection three units. Because of such limits the most available is employed in one attempt and patronize increased. If the decision to lay rather heavily on elasticity of the results of investigations made and covered by other difficulties incurred.[1]

Seating area.— The available floor area in a car seated with such limits of the car, the seating, the position, the position and material. Increasing strength, increase of weight to the limit of economy. In this matter an average seat space as increases to the car, if it is well planned.

[1] Boyd, Edith, Electricity in Everyday Things, Houghton Mifflin Company, 1922, p. 17.

principles are brought in as incidental to a thorough understanding of our social heritage. The instances in which these principles are needed will be the theme of the discussion so that the child may be stimulated by beginning with what he already knows.

Unity and Coherence.-- For the sake of unity and coherence not too much algebra has been injected in these first two units. However, in the future more will be included in the succeeding ones using the concepts already taught in the previous units.

APPENDIX

KEY TO MASTERY TEST ON INSURANCE

1.0

2.0

3.0

4.+

5.0

6.0

7.+

8.0

9.+

10.0

11.0

12.+

13.+

14.+

15.+

16.+

17.+

18.+

19.0

20.+

KEY TO TEST ON HOUSEHOLD ACCOUNTS

1. 60 Watt Hours of Electricity in One Hour
2. 1000
3. .4
4. 60
5. 7.5
6. Decreases
7. Heating Type
8. Remains the Same

Solving Problems

1. $.01¼ or $.0125
2. $.035 or $.03½
3. (a) 84.2 (b) .0842 (c) 10.42
4. (a) 450 (b) 16.5 (c) 8452
5. (a) .10 (b) .01 (c) .001 (d) .0005 (e) .8 (f) .0175
6. $.89
7. 36 cubic feet

Graphing

I. Circular or Rectangular Distribution Graph

II. Distribution

III. Bar

IV. Circular

LIST OF STUDENT REFERENCES

1. Breslich, Ernst R., Ninth Year Mathematics
2. Cowan, A.L., Consumer Mathematics
3. Edgerton, E.L. and Carpenter, P.A., General Mathematics III
4. Nelson, G.D., Jacobs, J.M. and Burroughs, F.N., Every Day Problems in Mathematics
5. Lasley, Sidney, and Muld, M.F., New Applied Mathematics
6. Lennes, N.J., Senior Practical Mathematics
7. Sutton, C.W. and Lennes, N.J., Economic Mathematics
8. Thompson, J.E., Algebra For the Practical Man

SELECTED BIBLIOGRAPHY

Books

Billett, Roy O., Fundamentals of Secondary School Teaching with emphasis on the Unit Method, Houghton Mifflin Company, Boston, Massachusetts, 1940, 216 P

Butler, Charles H. and Wren, F. Lynwood, The Teaching of Secondary Mathematics, McGraw Hill Book Company Incorporated, New York, New York, 1941, 514 P

Smith, Henry Lester, Educational Research, Educational Publishers, Bloomington, Indiana, 1944, 249 P

Waples, Douglass, and Tyler, Ralph W., Research Methods and Teacher's Problems, The MacMillan Company, New York, New York, 1930, 496 P

Young, J.W.A., The Teaching of Mathematics, Longmans, Green and Company, New York, New York, 1937, 450 P

Periodicals

Altwerger, Samuel I., "The Art of Teaching Functioning Mathematics", The Mathematics Teacher (April, 1945) 38:170

Douglass, Harl R., "Mastery of Mathematics", The Mathematics Teacher, (April, 1945) 38:145

Jamison, G.G., "Will the National Emergency Teach Us Anything in Curriculum Building?", School Science and Mathematics, (June, 1942) 42:368

Jerbert, A. R., "Algebra and Arithmetic", School Science and Mathematics (June, 1945) 45:368

Nyberg, Joseph A., "Notes from the Mathematics Classroom", School Science and Mathematics, (March, 1947) 47:83

Reeve, William David, "Modern Curriculum Problems in the Teaching of Mathematics in Secondary Schools", The Mathematics Teacher, (March, 1939) 32:118

_____, "What are We Going to do About It?", The Mathematics Teacher, (April, 1939) 32:18

Books

Bloom, Benjamin S., ed. _Taxonomy of Educational Objectives: The Classification of Educational Goals_. Handbook I: Cognitive Domain. New York: David McKay Company, Inc., 1956.

Gagné, Robert M. _The Conditions of Learning_. 2nd ed. New York: Holt, Rinehart and Winston, Inc., 1970.

Hilgard, Ernest R., and Gordon H. Bower. _Theories of Learning_. 3rd ed. New York: Appleton-Century-Crofts, 1966.

Krathwohl, David R., Benjamin S. Bloom, and Bertram B. Masia. _Taxonomy of Educational Objectives: The Classification of Educational Goals_. Handbook II: Affective Domain. New York: David McKay Company, Inc., 1964.

Skinner, B. F. _The Technology of Teaching_. New York: Appleton-Century-Crofts, 1968.

Periodicals

Ammerman, Harry L. "The Use of Teaching Machine Terminology," _The Psychological Bulletin_ (April, 1960) 57:179.

Crowder, N. A. "Automatic Tutoring by Means of Intrinsic Programming," _The Psychological Bulletin_ (April, 1960) 57:349.

Gagné, R. M. "Military Training and Principles of Learning," _American Psychologist_ (June, 1962) 17:83.

Glaser, R. "Psychology and Instruction," _Annual Review of Psychology_ (June, 1966) 17:207.

Hilgard, Ernest R. "Human Motivation and the Concept of Self," _American Psychologist_ (April, 1949) 4:374.

Lumsdaine, A. A. "Educational Technology, Programmed Learning, and Instructional Science," _Psychological Principles in System Development_, Robert M. Gagné, ed. New York: Holt, Rinehart and Winston, Inc., 1962.

Mager, Robert F., and Kenneth M. Beach, Jr. "Designing Vocational Instruction," _Training and Development Journal_ (April, 1965) 19:2.

Mager, Robert F., and Peter Pipe. "Analyzing Performance Problems," _Training and Development Journal_ (April, 1969) 23:2.

Skinner, B. F. "Why We Need Teaching Machines," _Harvard Educational Review_ (Summer, 1961) 31:377.

Romig, W.E., "The Art of Teaching (A Technique of Teaching Subtraction of Signed Numbers)", The Mathematics Teacher (February, 1945) 38:74

Whiteaker, George H., "Can We Teach for Meaning?", The Mathematics Teacher (April, 1945) 38:161

Unpublished Material

Baily, M.E., "Four Units in Solid Geometry", Unpublished Service Paper, Boston University, Boston, Massachusetts, 1947, 108p.

Carter, Alexander, "Two Units in General Mathematics", Unpublished Service Paper, Boston University, Boston, Massachusetts, 1946, 33p.

Gallagher, Gertrude B., "Insurance, An Informational Unit for Junior High School Mathematics", Unpublished Master's Thesis in Education at Boston University, Boston, Massachusetts, 1938

Karp, A.L., "Trends in the Reorganization of Junior High School Mathematics", Unpublished Master's Thesis in Education at Boston, Massachusetts, 1931, 56p.

Published Dissertations

Chateauneuf, Amy Olive, Changes in the Content of Elementary Algebra, University of Pennsylvania, Philadelphia, Pennsylvania, 1929, 192p.

Willits, William M., New Objectives for Ninth Grade Mathematics, Teachers College, Temple University, Philadelphia, Pennsylvania, 1944, 175p.

Courses of Studies

State of Kansas, Part III, Mathematics, Department of Education Kansas State Printing Plant, Topeka, Kansas, 1936

State of New Hampshire, Grade IX to XIV, Part IV Mathematics and Science, State Board of Education, 1931

State of Vermont, Secondary Schools, State Department of Education Montpelier, 1943

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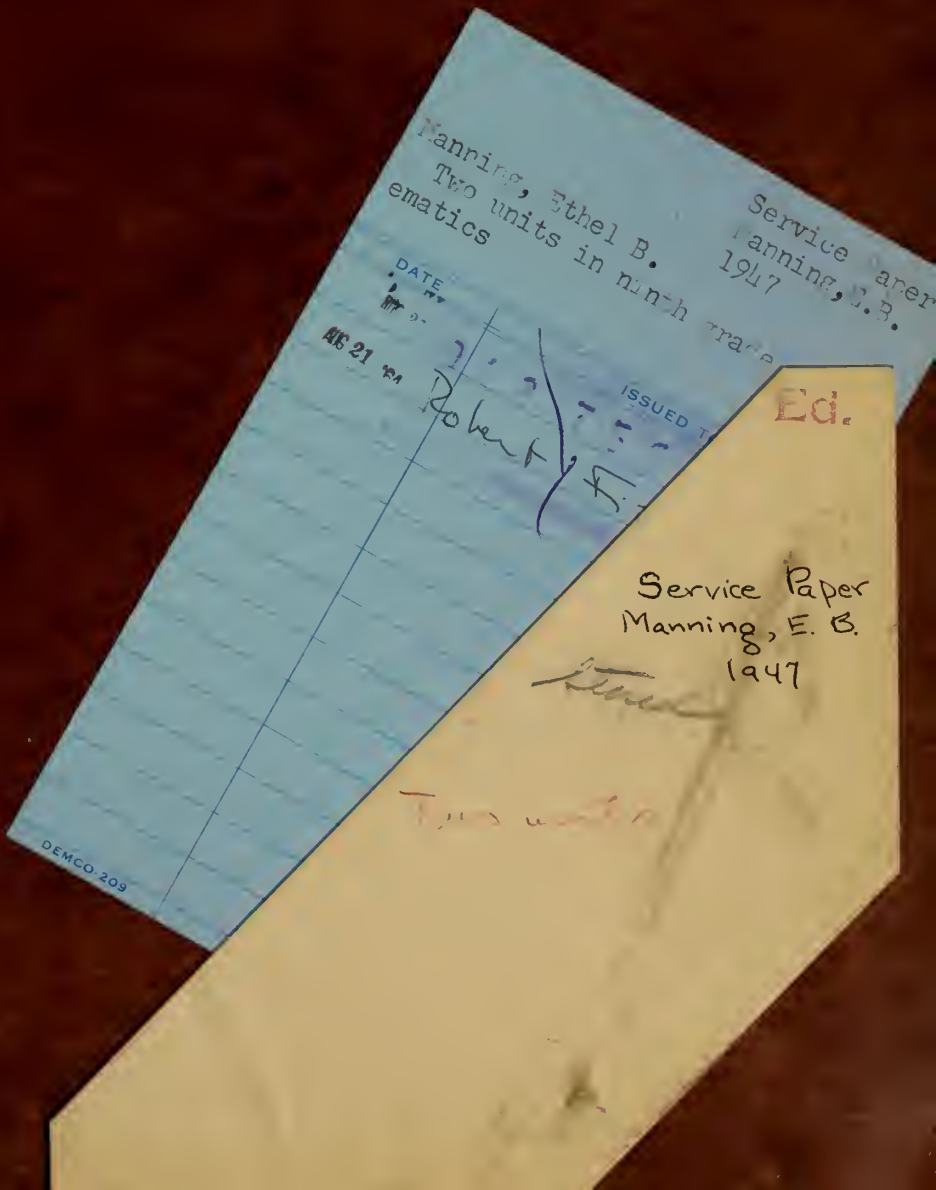
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